Stage 2

Medium Term Plans

September 2017
## Overview of Year

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<th>Number and Algebra</th>
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<td>5. Exploring Shape</td>
<td>6. Reasoning with Measures</td>
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<th>Number and Algebra</th>
<th>Statistics</th>
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<th>Geometry</th>
<th>Number and Algebra</th>
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<td>Unit</td>
<td>Approx Learning Hours</td>
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<td>------------------------------</td>
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</tbody>
</table>
| 1. Investigating Number Systems | 8                     | Reading and writing numbers in words and numerals to 100  
         |                               | Recognise place value; compare and order numbers |
| 2. Pattern Sniffing          | 8                     | Counting in 2s, 3s and 5s; recall and use times table facts for these  
         |                               | Arrange patterns; recognise odd and even numbers |
| 3. Solving Calculation Problems | 11                    | Recall addition and subtraction facts to 20; derive them to 100  
         |                               | Add and subtract numbers using objects and pictures up to 2d+2d  
         |                               | Show commutativity of addition (and non-commutativity of subtraction) |
| 4. Generalising Arithmetic   | 10                    | Solve addition and subtraction problems with 2-digit numbers  
         |                               | Solving missing number problems using inverses |
| 5. Exploring Shape           | 8                     | Identify and describe properties of 2D and 3D shapes |
| 6. Reasoning with Measures   | 8                     | Use £ and p symbols; find combinations of coins for a given total; solve simple addition and subtraction problems using money |
| 7. Discovering Equivalence  | 11                    | Recognise, find and name 1/3, ¼, 2/4, ¾ of a length, shape, set of objects or quantity  
         |                               | Write simple fractions and recognise equivalence of 2/4 and ½ |
| 8. Reasoning with Fractions  | 11                    | Calculate and write mathematical statements for multiplication and division  
         |                               | Show commutativity of multiplication (and non-commutativity of division)  
         |                               | Solve simple problems using multiplication and division |
| 9. Solving Number Problems   | 12                    | Calculate and write mathematical statements for multiplication and division  
         |                               | Show commutativity of multiplication (and non-commutativity of division)  
         |                               | Solve simple problems using multiplication and division |
| 10. Investigating Statistics | 6                     | Interpret and construct simple pictograms, tally charts, block diagrams and tables  
         |                               | Count, sort, total and compare categorical data |
| 11. Visualising Shape        | 8                     | Identify 2D shapes on the surface of 3D shapes |
| 12. Exploring Change         | 7                     | Sequence time intervals  
         |                               | Know number of minutes in an hour and hours in a day  
         |                               | Tell/show the time to the nearest 5 minutes |
| 13. Proportional Reasoning   | 7                     | Recall and use 2, 5 and 10 times tables  
         |                               | Calculate and write mathematical statements for multiplication and division  
         |                               | Show commutativity of multiplication (and non-commutativity of division) |
| 14. Describing Position      | 5                     | Describe position, direction and movement using mathematical language, distinguishing between straight line movement and rotation |
| 15. Measuring and Estimating | 8                     | Choose and use suitable units; compare and order lengths, masses and capacities |

Total = 117 hours ~ 24 weeks with 5 hours teaching per week
### Stage 2

#### Unit 1: Investigating Number Systems

This unit introduces the number systems and structures that we use at different levels of the curriculum. At KS1 children are working on the place value system of base 10 with the introduction of Roman Numerals as an example of an alternative system in KS2. Negative numbers and non-integers also come in at this stage and progress into KS3. At KS3 and KS4 we start to look at other ways of representing numbers, including standard form, inequality notation and so on.

<table>
<thead>
<tr>
<th>Prior Learning</th>
<th>Core Learning</th>
<th>Extension Learning</th>
</tr>
</thead>
<tbody>
<tr>
<td>➢ read and write numbers from 1 to 20 in numerals and words.</td>
<td>➢ read and write numbers to at least 100 in numerals and in words</td>
<td>➢ read and write numbers up to 1000 in numerals and in words</td>
</tr>
<tr>
<td>➢ identify and represent numbers using objects and pictorial representations including the number line, and use the language of: equal to, more than, less than (fewer), most, least</td>
<td>➢ recognise the place value of each digit in a two-digit number (tens, ones)</td>
<td>➢ recognise the place value of each digit in a three-digit number (hundreds, tens, ones)</td>
</tr>
<tr>
<td></td>
<td>➢ identify, represent and estimate numbers using different representations, including the number line</td>
<td>➢ identify, represent and estimate numbers using different representations</td>
</tr>
<tr>
<td></td>
<td>➢ compare and order numbers from 0 up to 100; use &lt;, &gt; and = signs</td>
<td>➢ solve number problems and practical problems involving these ideas</td>
</tr>
<tr>
<td></td>
<td>➢ use place value and number facts to solve problems</td>
<td></td>
</tr>
</tbody>
</table>

**Working towards expected standard**
- Demonstrate an understanding of place value, though may still need apparatus to support them (e.g. by stating the difference between 2 numbers i.e. 77 and 33 has a difference of 40 for the tens and 4 for the ones; by writing number statements such as 35<53 and 42>36)
- Read and write numbers correctly in numerals to 100 (e.g. can read and write 14 and 41 correctly)

**Working at expected standard**
- Partition two-digit numbers into different combinations of tens and ones. This may include using apparatus. (e.g. 23 is the same as 2 tens and 3 ones which is the same as 1 ten and 13 ones)

**Working at greater depth:**
- use multiplication facts to make deductions outside known multiplication facts (e.g. a pupil knows that multiples of 5 have one digit of 0 or 5 and uses this to reason that 18 × 5 cannot be 92 as it is not a multiple of 5).
### Exemplification

1. **a)** Write this number using numerals: Thirty-eighth  
   **b)** Write this number in words: 57

2. What is the value of the digit 7 in the number  
   i) 75  
   ii) 17

3. **a)** Represent 63 on this number line

   ![Number Line]

   **b)** Represent 34 using base 10

   **c)** State which number is shown here:

4. Write these numbers in order from smallest to largest: 48, 84, 40, 44, 80, 8

5. Here are some number cards
   **a)** Choose two cards to make the smallest possible 2 digit number

   **b)** Choose two cards to make the largest possible 2 digit number

<table>
<thead>
<tr>
<th>Representation</th>
<th>Fluency</th>
<th>Probing Questions</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Identifying and Representing Numbers</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>• Represent 2 digit numbers (loose and in column format) using:</td>
<td>1. Convert a given representation to a number (verbal or numerals)</td>
<td>Convince me that this is one more than 42</td>
</tr>
<tr>
<td>o Individual items e.g. counters, cubes</td>
<td>o single digits recap</td>
<td>What’s the same and what’s different? 1, 10, 100</td>
</tr>
<tr>
<td>o Individual items with 1:1 correspondence e.g. beadstrings</td>
<td>o multiples of ten recap (objects and number line)</td>
<td>Show me one more than sixty-four</td>
</tr>
<tr>
<td>o Separable to-scale tens and ones e.g. bundles of Straws, sticks of unifix</td>
<td>o tens and ones (separable e.g. straw bundles, multilink sticks, bead string)</td>
<td></td>
</tr>
<tr>
<td>o Inseparable to-scale tens and ones e.g. Numicon 10s and 1s, Base 10, Dienes rods</td>
<td>o tens and ones (inseparable e.g. Numicon, Dienes, base 10)</td>
<td></td>
</tr>
<tr>
<td>o Not-to-scale tens and ones e.g. place value counters, tens-ones abacus, money (10p and 1p coins –</td>
<td>o tens and ones (not to scale e.g. place value counters, money)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>o number line position</td>
<td></td>
</tr>
</tbody>
</table>

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@aetmaths
<table>
<thead>
<tr>
<th><strong>Partitioning</strong></th>
<th><strong>Reading and Writing Numbers</strong></th>
<th><strong>Converting Numbers</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>Representing numbers using tens and ones apparatus and then separating into different combinations e.g. 28 could be 20 + 8 or 10 + 18 or 25 + 3 and so on</td>
<td>Use (and make) a washing line to link numbers in words to numbers in numerals to images/representations e.g. numicon</td>
<td>Show me the number 63 in words</td>
</tr>
<tr>
<td>Digital sense of size of numbers up to 100 using paper strips and paperclips to position e.g. strip represents 0-100, where is 23? What if the strip now represents 0-50?</td>
<td>Use matching cards between words, numerals and representations of numbers</td>
<td>Show me how we can represent the number 35 using</td>
</tr>
<tr>
<td></td>
<td>Wherever numbers appear in the classroom, represent them in all three ways if possible e.g. on the clock</td>
<td>- unifix cubes</td>
</tr>
<tr>
<td></td>
<td>Where number formation is an issue, use tracing/stencil activities (with pens, in sand, on a tablet) to practice correct formation</td>
<td>- the beadstring</td>
</tr>
<tr>
<td></td>
<td>Explore the idea of unitisation using double-</td>
<td>- the dienes rods</td>
</tr>
<tr>
<td></td>
<td>- 4. Convert a number written in words to numerals and vice versa</td>
<td>- the cuisinaire rods</td>
</tr>
<tr>
<td></td>
<td>Recap single digits e.g. four or 7</td>
<td>- the numicon</td>
</tr>
<tr>
<td></td>
<td>Recap ‘Teen’ numbers e.g. fourteen or 19</td>
<td>- the hundred square</td>
</tr>
<tr>
<td></td>
<td>Recap multiples of ten e.g. eighty or 30</td>
<td>- the number line</td>
</tr>
<tr>
<td></td>
<td>Two digits e.g. eighty-four or 45</td>
<td>Convince me that 88 comes after 87</td>
</tr>
<tr>
<td></td>
<td></td>
<td>What's the same and what's different? 65, 56, 6, 5, 60, 50</td>
</tr>
<tr>
<td></td>
<td>5. Recognise matching numerals, words and representations</td>
<td>Show me where 62 would be on this blank paper strip that goes from 0-100?</td>
</tr>
<tr>
<td></td>
<td>Matching pairs</td>
<td>Always, Sometimes, Never?</td>
</tr>
<tr>
<td></td>
<td>Matching three or more items</td>
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</tbody>
</table>
sided counters to see how to use a different colour to represent a set amount of ones (heading towards the use of tens and ones as one of the easiest combinations)

<table>
<thead>
<tr>
<th>Exploring same, more, less</th>
<th>6. Compare two numbers to say which is greater, using &gt;, &lt; or = to notate</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Use apparatus and then visuals and then number cards (abstract) to explore which number is greater when comparing</td>
<td></td>
</tr>
<tr>
<td>• Compare number by building each using ones and then using ‘movable comparison sticks’ to show if they are equal or which is greater.</td>
<td></td>
</tr>
<tr>
<td>• Use a pan balance to help develop the concept of equal – this is particularly effective with Numicon, which is weighted so that it balances when of equal value.</td>
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<tr>
<td>• Use number cards to explore making different two-digit numbers and finding the smallest/largest</td>
<td></td>
</tr>
<tr>
<td>6. Compare two numbers to say which is greater, using &gt;, &lt; or = to notate</td>
<td></td>
</tr>
<tr>
<td>o Recap: two numbers up to 20</td>
<td></td>
</tr>
<tr>
<td>o One single-digit and one two-digit</td>
<td></td>
</tr>
<tr>
<td>o Two two-digit numbers (unrelated)</td>
<td></td>
</tr>
<tr>
<td>o Two two-digit numbers (similar digits)</td>
<td></td>
</tr>
<tr>
<td>o Mixture of representations/words/numerals</td>
<td></td>
</tr>
<tr>
<td>There are 10 numbers with a digit of 3 in them (up to 100).</td>
<td></td>
</tr>
<tr>
<td>Show me which is a greater 39 or 93?</td>
<td></td>
</tr>
<tr>
<td>Show me a number that could complete 54 &lt; ......</td>
<td></td>
</tr>
<tr>
<td>Convince me that 38 &lt; 83</td>
<td></td>
</tr>
<tr>
<td>What’s the same and what’s different? 41&gt;32, 76&lt; 85, 50 = 50, 54&lt;45</td>
<td></td>
</tr>
<tr>
<td>7. Order numbers from smallest to largest</td>
<td></td>
</tr>
<tr>
<td>o Order three numbers:</td>
<td></td>
</tr>
<tr>
<td>• Recap: numbers up to 20</td>
<td></td>
</tr>
<tr>
<td>• Two-digit numbers (unrelated)</td>
<td></td>
</tr>
<tr>
<td>• Two-digit numbers (similar digits)</td>
<td></td>
</tr>
<tr>
<td>o Order four or more numbers (as above)</td>
<td></td>
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<tr>
<td>What’s the same and what’s different? 49, 50, 51</td>
<td></td>
</tr>
<tr>
<td>8. Solve place value problems</td>
<td></td>
</tr>
<tr>
<td>o Find a number that lies between two given numbers (2-digits)</td>
<td></td>
</tr>
<tr>
<td>o Given selection of digits, produce the smallest and largest possible numbers from them</td>
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<tr>
<td>o Convert non-standard partitioning to standard e.g. sixty-fourteen</td>
<td></td>
</tr>
<tr>
<td>o Relate place value to money contexts (£1, 10p, 1p)</td>
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</tr>
<tr>
<td>Show me a number in between 45 and 76</td>
<td></td>
</tr>
<tr>
<td>Always, Sometimes, Never? There is only one possible answer to this gap question ...... &gt; 99</td>
<td></td>
</tr>
<tr>
<td>Always, Sometimes, Never? If you take a number and reverse its digits, you get a number that is bigger than you started with</td>
<td></td>
</tr>
</tbody>
</table>
### Further Extension

1. Write all the 2-digit numbers greater than 40 using these digits.
   - 2
   - 4
   - 6
   - 6

   How do you know you have them all? Prove it.

2. If you put 2 beads onto a tens/ones abacus you can make the numbers 2, 20 and 11.

   ![Abacus Diagram](image)

   Do the same with 3 beads. How many different numbers can you make?

3. Jo has £2.29.
   - She only has 1p coins, 10p coins and 1p coins.
   - How many of each coin does she have?
   - Can you suggest a different answer?

4. Here is part of a number square.
   - What is the largest number on the whole square?

<table>
<thead>
<tr>
<th></th>
<th>3</th>
<th>6</th>
<th>9</th>
<th>12</th>
<th>15</th>
</tr>
</thead>
<tbody>
<tr>
<td>18</td>
<td>21</td>
<td>24</td>
<td>27</td>
<td></td>
<td></td>
</tr>
<tr>
<td>33</td>
<td>36</td>
<td>39</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>48</td>
<td>51</td>
<td>5</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>63</td>
<td>66</td>
<td></td>
<td></td>
<td></td>
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</tr>
</tbody>
</table>

### Rich and Sophisticated Tasks

**Recognise the place value of each digit in a two-digit number (tens, ones)**

- NRICH: Snail One Hundred * G
- NRICH: Two-digit Targets * P
- NRICH: 6 Beads ** P

**Compare and order numbers from 0 up to 100; use <, > and = signs**

- NRICH: Domino Sequences * P
- NRICH: Domino Number Patterns ** P
- NRICH: Next Domino * P
- NRICH: 100 Square Jigsaw * G
- NRICH: That Number Square! * P I

**Use place value and number facts to solve problems**

- NRICH: I Like … * G
- NRICH: Light the Lights *** G
- NRICH: Largest Even * G P
- NRICH: Round the Two Dice * P I
5.

Use < > and = signs to make these number sentences correct.

<table>
<thead>
<tr>
<th>3 tens and 2 ones</th>
<th>2 tens 12 ones</th>
</tr>
</thead>
<tbody>
<tr>
<td>4 tens and 3 ones</td>
<td>3 tens 14 ones</td>
</tr>
<tr>
<td>5 tens and 4 ones</td>
<td>4 tens 11 ones</td>
</tr>
</tbody>
</table>

6.

Place 47 on each of these empty number lines.

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
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<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>10</td>
<td>20</td>
</tr>
<tr>
<td>30</td>
<td>40</td>
<td>50</td>
</tr>
<tr>
<td>60</td>
<td>70</td>
<td>80</td>
</tr>
<tr>
<td>90</td>
<td>100</td>
<td>110</td>
</tr>
</tbody>
</table>

### Misconceptions

- Children confuse ‘teen’ and ‘ty’ numbers e.g. fourteen and forty
- Children may use a 0 placeholder incorrectly, e.g. recording thirty-four as 304.
- Additionally some children incorrectly read out the ones before the tens e.g. reading 23 as either thirty-two or sometimes three (and) twenty or two three rather than twenty-three.
- Children confuse the meaning of < and >, finding it hard to tell which is which.
- Children confuse the order of tens and ones and so will think that fifty-eight can be represented as 85
- Children think of every number separately e.g. 23 is 23 ones and they cannot see it also as 2 tens and three ones
- Children also cannot see past standard partitioning e.g. they do not recognise eighty-thirteen as the same as ninety-three
- Children interpret = as 'the answer is' rather than 'is worth the same as' - make sure you model this carefully and avoid implying that it simply starts the answer!

### Teacher Guidance and Notes

- You need to develop a strong understanding of place value so that children see that they only need to actually learn the numbers 0-9 because all other numbers just use these in different positions.
- When teaching place value use practical resources to expand on different base representations to emphasise the unitised structure of number i.e. 31 as 3 ten rods and 1 unit/ ones in Base 10 (the list of suggested apparatus is shown above and the intention is for all children to experience all of these representations).
- It is important that children develop their number sense here - they should be able to place numbers on a blank number line including where the scale changes. Try taking a blank paper strip as a scale from 0-100 and asking children to place 24 on it. Then change the scale to 1-50 and ask them to do the same thing - they should be developing the ability to change the placement based on the scale.
<table>
<thead>
<tr>
<th>Key Assessment Checklist</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. I can read and write numerals and words from 10 to 50.</td>
</tr>
<tr>
<td>2. I can read and write numerals and words from 50 to 100.</td>
</tr>
<tr>
<td>3. I can say the number that is one more or less than a given number to 100.</td>
</tr>
<tr>
<td>4. I can understand the value of each digit in a 2-digit number and partition the number as tens and ones</td>
</tr>
<tr>
<td>5. I can represent any number up to 100 using: unifix cubes oe; a number line; a hundred square; numicon; cuisenaire rods; Dienes; a tens and ones abacus; a bead string, place value counters</td>
</tr>
<tr>
<td>6. I can solve problems using my knowledge of number facts and place value.</td>
</tr>
<tr>
<td>7. I can compare two numbers up 100 and use the signs &lt;, &gt; (and =) to show this comparison.</td>
</tr>
<tr>
<td>8. I can compare up to three numbers or amounts up to 100, order them and say which is the most and which is the least.</td>
</tr>
</tbody>
</table>
## Stage 2

### Unit 2: Pattern Sniffing

8 learning hours

This unit explores pattern from the early stages of counting and then counting in 2s, 5s, and 10s up to the more formal study of sequences. This sequence work progresses through linear sequences up to quadratic, other polynomial and geometric for the most able older students. For children in KS1, this unit is heavily linked to the following one in terms of relating counting to reading and writing numbers.

Also in this unit children and students begin to study the properties of numbers and to hone their conjecture and justification skills as they explore odd/even numbers, factors, multiples and primes before moving onto indices and their laws.

### Prior Learning

- count to and across 100, forwards and backwards, beginning with 0 or 1, or from any given number
- count, read and write numbers to 100 in numerals; count in multiples of twos, fives and tens

### Core Learning

- count in steps of 2, 3, and 5 from 0, and in tens from any number, forward and backward
- order and arrange combinations of mathematical objects in patterns and sequences
- **begin to recall** and use multiplication and division facts for the 2, 5 and 10 multiplication tables, including recognising odd and even numbers

**Working towards expected standard**

- Count in twos, fives and tens from 0 and use counting strategies to solve problems e.g. count the number of chairs in a diagram where the chairs are organised in 7 rows of 5 by counting in 5s

**Working at expected standard**

- recall and use multiplication and division facts for the 2, 5 and 10 multiplication tables to solve simple problems, demonstrating an understanding of commutativity as necessary (e.g. knowing they can make 7 groups of 5 from 35 blocks and writing 35 ÷ 5 = 7; sharing 40 cherries between 10 people and writing 40 ÷ 10 = 4; stating the total value of six 5p coins)

**Working at greater depth:**

- use multiplication facts to make deductions outside known multiplication facts (e.g. a pupil knows that multiples of 5 have one digit of 0 or 5 and uses this to reason that 18 × 5 cannot be 92 as it is not a multiple of 5).

### Extension Learning

- count from 0 in multiples of 4, 8, 50 and 100;
- find 10 or 100 more or less than a given number
- recall and use multiplication and division facts for the 3, 4 and 8 multiplication tables
### Exemplification

1. **a)** Write the next three numbers in the number patterns:
   - i) 0, 3, 6, 9, 12, 15, ...., ...., ...., ...., ....,
   - ii) 0, 5, 10, 15, 20, 25, 30, 35, ...., ...., ....,

   **b)** Count on in 10s from 37: ....... ....... .......

2. Continue this pattern with Dienes rods and ones:

![Dienes rods and ones](image)

3. **a)** Complete the missing numbers in the number sentences:
   - i) 2 x 8 = ........ ii) 5 x ........ = 35 iii) 60 ÷ ........ = 6 iv) ........ ÷ 2 = 7

   **b)** Which of these numbers are even? Circle them:

   13  56  73  84  100

### Vocabulary

- count from...
- count in...
- forwards/backwards
- pattern
- number pattern
- tens and ones
- place value
- missing number
- times table
- multiplication
- division
- array
- row
- column
- groups
- fact family
- odd
- even

### Representation

**Counting:**
- Counting aloud using marked number lines, number tracks, 100 squares and counting sticks to prompt (Give children some apparatus to use alongside that which you are using with the group e.g. their own number track or their hundred square)
- Using a counting stick to start counting at a new value (forwards or backwards)
- Represent counting in 2s, 3s 5s and 10s using repeated addition with:
  - Numicon pieces
  - Counters on a blank track
  - Counters in groups of 2, 5 or 10
  - Bead string
  - Placing counter on/Colouring in 100-square

#### Fluency

1. **Count from 0 in steps of 2, 3 and 5**
   - work out the steps using repeated addition
   - work out some steps using doubling skills
   - count from 0 up to 10<sup>th</sup> multiple of 2, 3 and 5 with concrete/visual aid
   - count from 0 up to 10<sup>th</sup> multiple of 2, 3 and 5 without concrete/visual aid
   - count from 0 beyond 10<sup>th</sup> multiple of 2, 3 and 5

2. **Count in 10s**
   - from 0 (forwards)
   - from 100 (backwards)
   - beyond 100 from a multiple of 10 (forwards or backwards)

#### Probing Questions

- Show me how we count in 10s on a hundred square
- Convince me that 31 is ten more than 21
- Convince me that 55 comes next 35, 40,
### Money (2ps, 5ps and 10ps)
- Represent counting in 10s (from any number) using a hundred square to discover the link between jumping a row and counting on (or back) ten
- Use a counting stick to represent the first ten multiples of 2, 3, 5 and 10 – explore which values can be found by doubling.

### Pattern
- Building patterns from shapes, number equipment (Numicon, dienes etc) and other objects (e.g. animals, pencils) and beginning to describe the rule or pattern
- Finding the next item in a practical pattern
- Given a rule, building a pattern of objects

### Times Table Facts
- Represent the 2 times table using a counting stick for the first ten multiples (see above)
- Represent a times table multiplication calculation using repeated addition in multiple ways:
  - e.g. $2 \times 6$ as:
    - 6 groups of 2 objects
    - 6 Numicon 2s
    - 6 lots of 2-rods (Cuisenaire)
    - as an array made of 6 rows of 2 counters/dots
- to begin to deduce the related facts i.e. $2 \times 6 = 12$; $6 \times 2 = 12$; $12 \div 2 = 6$ and $12 \div 6 = 2$ (more on this in Unit 9)
### Odds and Evens

- Use Numicon to explore odd and even numbers by building a number and then sorting out into representations with ‘a bit that sticks out’ and those that don’t.
- (You can do a similar thing with cubes i.e. which numbers of cubes can be arranged into an array with 2 columns and no leftovers?)

### Times Table Facts

- Represent the 5 and 10 times tables using a counting stick for the first ten multiples (see above)
- Represent a times table multiplication calculation using repeated addition in multiple ways:
  - e.g. $5 \times 6$ as:
    - 6 groups of 5 objects
    - 6 numicon 5s
    - 6 lots of 5-rods (Cuisenaire)
    - as an array made of 6 rows of 5 counters/dots

  To begin to deduce the related facts i.e. $5 \times 6 = 30$; $6 \times 5 = 30$; $30 \div 5 = 6$ and $30 \div 6 = 5$ (more on this in Unit 9)

### 5. Recognise odd and even numbers

- Say if a number is odd or even using apparatus
- Say if an object is odd or even using the numerals
- Find an odd or even number of a given size e.g. greater than 30 or between 50 and 60

### 6. Find and begin to recall times table division facts (10s, 5s)

- By representing the calculation concretely to deduce the answer
- By representing the calculation visually to deduce the answer
- By relating the calculation to another known calculation and counting on/back or doubling etc
- By beginning to recall key facts

<p>| Show me an odd number bigger than 20 |
| Show me an even number between 40 and 50 |
| Convince me that 53 is an odd number |
| What's the same and what's different? 1,3,5,8 |
| What's the same and what's different? 2,12,24,36? |
| Always, Sometimes, Never? Even numbers always end in 0,2,4,6,8. Even numbers can always be shared. Odd numbers always end in 1,3,5,7,9. |
| Convince me that the answer to $4 \times 5$ is an even number |
| What's the same and what's different? 25,30,36,40 |</p>
<table>
<thead>
<tr>
<th>Further Extension</th>
<th>Rich and Sophisticated Tasks</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>1.</strong></td>
<td>Count in steps of 2, 3, and 5 from 0, and in tens from any number, forward or backward</td>
</tr>
<tr>
<td>Captain Conjecture says, 'When I count in tens from any number the units digit stays the same.' Do you agree? Explain your reasoning.</td>
<td><strong>NRICH:</strong> [Buzzy Bee] * P [Five Steps to 50] * I</td>
</tr>
<tr>
<td><strong>2.</strong></td>
<td>Odd and Even Numbers</td>
</tr>
<tr>
<td><img src="image" alt="Pattern" /></td>
<td><strong>NRICH:</strong> [How Odd] ** [Even and Odd] *</td>
</tr>
<tr>
<td>Fill in the missing shape to complete the pattern. If the pattern continued what would the tenth shape be? Explain your reasoning.</td>
<td><strong>Patterns</strong></td>
</tr>
<tr>
<td><strong>3.</strong></td>
<td><strong>NRICH:</strong> [Poly Plug Pattern] *</td>
</tr>
<tr>
<td>Amy thinks of a number. Her number: ■ is an even number ■ is between 20 and 25 ■ has two different digits. What is her number? Explain your reasoning.</td>
<td></td>
</tr>
<tr>
<td><strong>4.</strong></td>
<td>Think of an even number that is more than 30 and less than 50. And another. Can you find them all? How many are there? Explain your reasoning.</td>
</tr>
</tbody>
</table>
### Misconceptions

<table>
<thead>
<tr>
<th>Pupils forget to include 0 when counting</th>
</tr>
</thead>
<tbody>
<tr>
<td>When counting in 10s, children forget what to do after 'ninety-something' as they bridge 100</td>
</tr>
<tr>
<td>Children change the rule in a pattern so that it doesn't flow throughout - they also interpret a pattern as 'pretty' rather than a sequence with a rule....</td>
</tr>
<tr>
<td>Children think that any number with a 3 in it will be odd e.g. 34.</td>
</tr>
<tr>
<td>Additionally they believe that any number ending in 3 is a multiple of 3!</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Teacher Guidance and Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>- This unit focuses on counting and the beginnings of the use of repeated addition to underpin multiplication. Note that multiplication is fully explored in Unit 9 and 13 and therefore you should avoid delving too deep into commutativity etc at this stage to find fact families. Instead concentrate on representing the 'counting in 2s' process as repeated addition of 2 and on saying this as '2 multiplied by ... is ....'. The other facts will follow later for most children.</td>
</tr>
<tr>
<td>- Define an even number as a number that is an exact multiple of 2 rather than 'one ending in 2, 4, 6, 8, 0' so that children are encouraged to explore what the number properties actually are to decide. Allow them to 'discover' the shortcuts of using the numerals rather than telling them the rule.</td>
</tr>
</tbody>
</table>

### Key Assessment Checklist

1. I can count in steps of 2s, and 5s from 0. I can talk about the number patterns.
2. I can count in steps of 3s from 0.
3. I can count forwards and backwards in 10s from any given number.
4. I can recognise odd numbers.
5. I can recognise even numbers.
6. I can use numicon to make mathematical patterns.
7. I can use mathematical resources to generate sequences.
8. I can begin to recall multiplication and division facts for 2s, 5s and 10s.
## Stage 2

### 11 learning hours

This unit explores the concepts of addition and subtraction at KS1 building to wider arithmetic skills including multiplication at KS2. It is strongly recommended that teachers plan this unit for KS1/KS2 with direct reference to the calculation policy! At KS3 students are developing calculation into its more general sense to explore order of operations, exact calculation with surds and standard form (which have been introduced in Inv Number Systems briefly) as well developing their skills in generalising calculation to algebraic formulae.

### Prior Learning

- Represent and use number bonds and related subtraction facts within 20
- Add and subtract one-digit and two-digit numbers to 20, including zero
- Read, write and interpret mathematical statements involving addition (+), subtraction (−) and equals (=) signs

### Core Learning

- Recall and use addition and subtraction facts to 20 fluently, and derive and use related facts up to 100
- Add and subtract numbers using concrete objects, pictorial representations, and mentally, including:
  - A two-digit number and ones
  - A two-digit number and tens
  - Two two-digit numbers
  - Adding three one-digit numbers
- Show that addition of two numbers can be done in any order (commutative) and subtraction of one number from another cannot

### Learning Leads to....

- Add and subtract numbers mentally, including:
  - A three-digit number and ones
  - A three-digit number and tens
  - A three-digit number and hundreds
- Add and subtract numbers with up to three digits, using formal written methods of columnar addition and subtraction estimate the answer to a calculation and use inverse operations to check answers

### Working towards

- Use number bonds and related subtraction facts within 20 (e.g. 18 = 9 + 9; 15 = 6 + 9)
- Add and subtract a 2-digit number and ones and a 2-digit number and tens when no regrouping is required (e.g. 23+5; 46+20), demonstrate their method using concrete apparatus or pictorial representations

### Working at expected standard

- Add 2 two-digit numbers within 100 (e.g. 48 + 35) and demonstrate their method using concrete apparatus or pictorial representations
- Use estimation to check that their answers to a calculation are reasonable (e.g. knowing that 48 + 35 will be less than 100)
- Subtract mentally a two-digit number from another two-digit number when there is no regrouping required (e.g. 74 – 33)

### Working at greater depth:

- Reason about addition (e.g. pupil can reason that the sum of 3 odd numbers will always be odd)
- Work out mental calculations where regrouping is required (e.g. 52 – 27 or 91-73)
### Exemplification

1. Find the value of the ★ in each statement:
   a) ★ + 5 = 13  
   b) 16 = ★ + ★  
   c) 18 - 7 = ★  
   d) 15 = 19 - ★

   e) Use the cards to complete the number facts:
   
   ![Number cards]

   

2. Calculate (using objects and images to support as required):
   
   a) 78 + 7  
   b) 56 - 3  
   c) 63 - 20  
   d) 48 + 50  
   e) 35 + 28  
   f) 67 - 25  
   g) 3 + 8 + 4  
   h) 6 + 4 + 9

3. True or False?
   a) 7 + 6 has the same answer as 6 + 7  
   b) 7 – 6 has the same answer as 6 – 7

### Representation

<table>
<thead>
<tr>
<th><strong>Addition</strong></th>
<th><strong>Fluency</strong></th>
<th><strong>Probing Questions</strong></th>
</tr>
</thead>
</table>
| • Representing numbers using tens and ones equipment then combining and finding the total value (aggregation) (exchanging ten 1s for one 10 as required when bridging)  
  Progression of tens and ones equipment:  
  o separable: bundles of straws or sticks of multilink cubes  
  o inseparable: Dienes rods or Base 10 or Numicon 10s and 1s  
  o not to scale: place value counters or money  
  • Representing addition as counting or jumping on (augmentation) using:  
    o a number track  
    o a bead string (reading the answer using 10s and 1s) | 1. Add a two digit number and ones  
  • 2-digit number (not ending in 9) + 1  
  • 2-digit number (ending in 9) + 1  
  • 2-digit number (not ending in 8 or 9) + 2  
  • 2-digit number (ending in 8 or 9) + 2  
  • 2-digit number + 3, 4, 5,...,(not crossing next ten) e.g. 34 + 3  
  • 2-digit number + 3, 4, 5,...,(crossing next ten) e.g. 34 + 9  
  • 2-digit number + 3, 4, 5,... (making exact multiple of 10) e.g. 34 + 6  
  • 2-digit number + 0 | Show me two numbers with a sum of 20  
Show me two numbers with a sum of 17  
Always, Sometimes, Never?  
A two-digit number add a one-digit number gives a two-digit number |
| 2. Add a two digit number and tens  
  • 2-digit multiple of 10 + 10 e.g. 30 + 10  
  • 2-digit number +10  
  • 2-digit multiple of 10 + 20 e.g. 30 + 20  
  • 2-digit number + 20 | Convince me that if I add ten to a number I will get the same digit in my 1s column as a I started with  
Always, Sometimes, Never? |
### Addition

- Representing addition word problems using the bar model
  - 2-digit multiple of 10 + 30, 40, 50, .... (answer ≤ 100)
  - 2-digit number + 30, 40, 50, .... (answer ≤ 100)

### Word Problems

3. **Add a two-digit number and a two-digit number (answer ≤ 100)**
   - two-digit + two-digit (not crossing a ten)
   - two-digit + two-digit (crossing a ten)
   - two-digit + two-digit (answer a multiple of ten)

4. **Subtract ones from a two-digit number**
   - 2-digit number (not ending in 0) - 1
   - 2-digit number (ending in 0) - 1
   - 2-digit number (not ending 0 or 1) - 2
   - 2-digit number (ending in 0 or 1) - 2
   - 2-digit number - 3, 4, 5,......(not crossing next ten) e.g. 34 - 3
   - 2-digit number - 3, 4, 5,......(crossing next ten) e.g. 34 - 9
   - 2-digit number - 3, 4, 5, ... (making exact multiple of 10) e.g. 34 - 4
   - 2-digit number - 0

5. **Subtract tens from a two-digit number**
   - 2-digit multiple of 10 - 10 e.g. 80 - 10
   - 2-digit number - 10
   - 2-digit multiple of 10 + 20 e.g. 80 - 20
   - 2-digit number - 20
   - 2-digit multiple of 10 - 30, 40, 50, .... e.g. 70 - 40
   - 2-digit number - 30, 40, 50, .... e.g. 73 - 40

### Subtraction

- Representing subtraction as counting or jumping back (reduction) using:
  - a number track
  - a bead string (reading the answer using 10s and 1s)
  - a hundred square (just a number track split into rows – jumping in 10s and 1s)
  - a marked number line (jumping in 10s and 1s)
  - an unmarked number line
- Representing subtraction as a comparative difference between two sets of objects using:
  - counters
  - beadstrings
  - Numicon tens and ones (and other)

### Word Problems

3. **Add a two-digit number and a two-digit number (answer ≤ 100)**
   - two-digit + two-digit (not crossing a ten)
   - two-digit + two-digit (crossing a ten)
   - two-digit + two-digit (answer a multiple of ten)

4. **Subtract ones from a two-digit number**
   - 2-digit number (not ending in 0) - 1
   - 2-digit number (ending in 0) - 1
   - 2-digit number (not ending 0 or 1) - 2
   - 2-digit number (ending in 0 or 1) - 2
   - 2-digit number - 3, 4, 5,......(not crossing next ten) e.g. 34 - 3
   - 2-digit number - 3, 4, 5,......(crossing next ten) e.g. 34 - 9
   - 2-digit number - 3, 4, 5, ... (making exact multiple of 10) e.g. 34 - 4
   - 2-digit number - 0

5. **Subtract tens from a two-digit number**
   - 2-digit multiple of 10 - 10 e.g. 80 - 10
   - 2-digit number - 10
   - 2-digit multiple of 10 + 20 e.g. 80 - 20
   - 2-digit number - 20
   - 2-digit multiple of 10 - 30, 40, 50, .... e.g. 70 - 40
   - 2-digit number - 30, 40, 50, .... e.g. 73 - 40

6. **Subtract a two-digit number from a two-digit number**
   - two-digit - two-digit (not crossing a ten)
   - two-digit - two-digit (answer a multiple of ten)
   - two-digit - two-digit (crossing a ten)

---

**Subtraction**

- Representing first number using tens and ones equipment then removing or taking away the second number and finding the resulting value (partitioning) (exchanging one 10 for ten 1s as required when bridging)
- Progression of tens and ones equipment:
  - separable: bundles of straws or sticks of multilink cubes
  - inseparable: Dienes rods or Base 10 or Numicon 10s and 1s
  - not to scale: place value counters or money
- Representing subtraction as counting or jumping back (reduction) using:
  - a number track
  - a bead string (reading the answer using 10s and 1s)
  - a hundred square (just a number track split into rows – jumping in 10s and 1s)
  - a marked number line (jumping in 10s and 1s)
  - an unmarked number line
- Representing subtraction as a comparative difference between two sets of objects using:
  - counters
  - beadstrings
  - Numicon tens and ones (and other)
### Commutativity
- Exploring whether \(a+b\) gives the same answer as \(b+a\) using different equipment (Numicon, Dienes, bead strings, counters)
- Exploring whether \(a - b\) gives the same answer as \(b - a\) using different equipment (discovering that when you try to take a larger number from a smaller one you ‘run out’ of objects to take away)

### Fact Families
- Drawing a bar model to represent the four related calculations
  - Example: \(12 + 5 = 17; 5 + 12 = 17; 17 - 5 = 12\) and \(17 - 12 = 5\)

### Number Bonds
- Exploring the pattern of bonds with the same answer e.g. using two different colour counters to show the different ways that you can make 11 by adding two numbers. Double-sided counters can work well for this.
- Alternatively use a bead string to see how you can partition a number in different ways e.g. count out 13 beads and find all the different pairs of numbers that add to 13 by splitting up your beads differently.
- Using a number line/counting stick to find pairs of numbers with the same total – a string number line is excellent for this as you can hold it up to show pairs of numbers with the same total hanging together

### State whether two addition or subtraction calculations have the same result
- Addition of single digits
- Addition of two-digit numbers
- Subtraction of single digits
- Subtraction of two-digit numbers
- General principle for addition
- General principle for subtraction

### Find fact family given one addition/subtraction fact
- Single digit addition fact given e.g. \(6 + 3 = 9\) (so find \(3 + 6 = 9\), \(9 - 6 = 3\) and \(9 - 3 = 6\))
- Two-digit addition fact given e.g. \(26 + 37 = 63\)
- Single digit subtraction fact given e.g. \(8 - 3 = 5\)
- Two-digit subtraction fact given e.g. \(43 - 28 = 15\)

### Recall number bonds
- Bonds to 10
- Bonds within 10 (i.e. to 9, 8, 7, 6, 5, 4, 3)
- Bonds to 20
- Bonds within 20 (i.e. to 11, 12, 13, 14, 15, 16, 17, 18, 19)

### Show me all the pairs of numbers with a sum of 13
- What’s the same and what’s different? 6+4, 5+5, 3+8, 1+9

### Add three one-digit numbers
- Answer < 10
- Example: where two numbers bond to 10 e.g. \(6 + 4 + 7\) or \(7 + 8 + 3\)
- Example: where two numbers bond to 11 e.g. \(8 + 3 + 5\)
- Examples where two numbers of the three bond to 12, 13, etc.

### Always, Sometimes, Never?
- A two-digit number subtract a two-digit number gives a two-digit number
- Convince me that the answer to \(45 + 28\) will be the same as the answer to \(28 + 45\)
- Convince me that the answer to \(7 + 9 + 6\) will be the same as \(6 + 9 + 7\) AND \(9 + 7 + 6\).
- What’s the same and what’s different? addition and subtraction

### Show me if I know that \(7 + 4 = 11\), what else I know?

### Show me three one digit numbers with a sum of 15
- Convince me that there are 5 more sums with the same numbers that give the same answer as \(4 + 6 + 8\)
**Further Extension**

1. Find different possibilities.

   \[
   \begin{array}{c}
   \square + \square = 50 \\
   50 - \square = \square
   \end{array}
   \]

2. Complete the calculations.

   \[
   \begin{array}{c}
   30 + 40 + \square = 100 \\
   40 + \square + 20 = 100 \\
   36 + 44 + \square = 100 \\
   36 + 54 + \square = 100 \\
   47 + \square + 20 = 100 \\
   47 + \square + 30 = 100
   \end{array}
   \]

**Rich and Sophisticated Tasks**

Recall and use addition and subtraction facts to 20 fluently, and derive and use related facts up to 100

- **NRICH:** Number Round Up *** G P
- **NRICH:** 4 Dom *** G P
- **NRICH:** Strike it Out * G

Add and subtract numbers using concrete objects, pictorial representations, and mentally, including:

- a two-digit number and ones
- a two-digit number and tens
- two two-digit numbers
- adding three one-digit numbers

- **NRICH:** Cuisenaire Environment * G
- **NRICH:** Jumping Squares ** G
- **NRICH:** Number Balance ** I
- **NRICH:** Unit Differences * P I
- **NRICH:** Dicey Addition * G
3. If each peg on the coat hanger has a value of 10, find three ways to partition the pegs to make the number sentences complete.

\[ \square + \square + \square = \square \]
\[ \square + \square + \square = \square \]
\[ \square + \square + \square = \square \]

What is the total of each addition sentence?
Will the total always be the same?
Explain your reasoning.

4. Fill in the missing numbers. What do you notice?

<table>
<thead>
<tr>
<th></th>
<th>27</th>
<th>12</th>
<th>15</th>
</tr>
</thead>
<tbody>
<tr>
<td>15</td>
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<tr>
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<tr>
<th></th>
<th>13</th>
<th>14</th>
<th>57</th>
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</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>15</td>
<td></td>
</tr>
</tbody>
</table>

5. Captain Conjecture says, ‘An odd number + an odd number + an odd number = an even number.’
Is this sometimes, always or never true?

Explain your reasoning.

*Concrete resources might help pupils to explain their reasoning.*
<table>
<thead>
<tr>
<th>Misconceptions</th>
<th>Teacher Guidance and Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Children struggle to interpret whether to add or subtract from the language</td>
<td>• The aim of this unit for these children is to explore how we can more</td>
</tr>
<tr>
<td>used. Children can find 'How many more/less?' particularly troublesome as it</td>
<td>efficiently add up two numbers beyond the known methods of counting all</td>
</tr>
<tr>
<td>relates to ordinal values of numbers and relationships.</td>
<td>or counting on. Therefore, many of the techniques at Stage 2 rely on</td>
</tr>
<tr>
<td>Children struggle to add two digit numbers when their place value</td>
<td>some understanding of place value and so this must be secure first. You should make</td>
</tr>
<tr>
<td>understanding is weak. If they do not read a number like '52' as 5 tens and 2</td>
<td>reference to the calculation policy specifically when teaching this unit as the range of</td>
</tr>
<tr>
<td>ones then they struggle to combine the ones and the tens from two numbers</td>
<td>equipment and the language are modelled there.</td>
</tr>
<tr>
<td>appropriately.</td>
<td>• The Representing section shows the different ways we can carry out</td>
</tr>
<tr>
<td>Children often do not see difference as a representation of subtraction</td>
<td>addition – in general the place value approaches are leading towards</td>
</tr>
<tr>
<td>because take away is emphasised so much. They need to see subtraction</td>
<td>(eventually!) written column methods while the counting on/number line</td>
</tr>
<tr>
<td>represented in this way also to challenge this.</td>
<td>approaches are leading towards mental methods. Both are important but</td>
</tr>
<tr>
<td>When subtracting, children will sometimes subtract the larger number from</td>
<td>they are not interchangeable so think carefully about whether you are focusing on</td>
</tr>
<tr>
<td>the smaller initially. They may also fail to exchange a ten for 10 ones</td>
<td>written or mental calculation to decide which representation is</td>
</tr>
<tr>
<td>when necessary and simply find the difference between the digits in the</td>
<td>best in a given lesson/skill/concept. Both need to be covered in the unit</td>
</tr>
<tr>
<td>column e.g. 43 - 27 - they may find the difference between the 3 and the 7</td>
<td>but not necessarily at the same time!</td>
</tr>
<tr>
<td>in the ones column as 4 rather than exchanging a ten for 10 ones to get</td>
<td>• When learning about addition that bridges 10, children need to literally and</td>
</tr>
<tr>
<td>13 - 7.</td>
<td>practically exchange 10 ones to enable them to carry out the</td>
</tr>
<tr>
<td>When exchanging does happen, children sometimes forget to 'remove' the 10</td>
<td>subtraction of the 1s. They can eventually move on to doing this with more</td>
</tr>
<tr>
<td>and so end up with ten too many in the answer.</td>
<td>abstract equipment such as place value counters.</td>
</tr>
<tr>
<td>When counting on, children may start counting the first extra number using</td>
<td>• To begin to embed the written routines of the calculation policy, it is</td>
</tr>
<tr>
<td>the start number itself, rather than counting the next number (thus they</td>
<td>advised that children work in pairs with one child manipulating the</td>
</tr>
<tr>
<td>end up with one less than the real answer. The same is true for subtraction</td>
<td>equipment and saying what they are doing aloud while the other child records the</td>
</tr>
<tr>
<td>in reverse. Look out for weak counting in general, which will hold this unit</td>
<td>calculation using the column method so that they learn that</td>
</tr>
<tr>
<td>back a great deal.</td>
<td>the column method is just a written representation of the practical process</td>
</tr>
<tr>
<td>Children forget to include 0 when counting and using number lines – they</td>
<td>(rather than a 'different' method).</td>
</tr>
<tr>
<td>often see the distance between 0 and 1 as being different to that between</td>
<td>• It is important to let children explore whether order matters when we add</td>
</tr>
<tr>
<td>other whole numbers (wrongly!).</td>
<td>and subtract - this is a great opportunity for some early investigation. Does</td>
</tr>
<tr>
<td>When working with number facts and bonds, children sometime realise there</td>
<td>44 + 27 give you the same answer as 27 + 44? etc. This knowledge saves a</td>
</tr>
<tr>
<td>is a connection e.g. 3 + 4 = 7 but then incorrectly rearrange this to make</td>
<td>lot of time then in recalling number bonds and facts as only half need to be</td>
</tr>
<tr>
<td>a false second fact e.g. 4 + 7 = 3. This is particularly true with</td>
<td>learnt!</td>
</tr>
<tr>
<td>subtraction facts, where children struggle to place the numbers in a correct</td>
<td>• The pitch of this unit is numbers up to 100, but of course these ideas an</td>
</tr>
<tr>
<td>order.</td>
<td>be extended beyond 100 for those children who are confident working with</td>
</tr>
<tr>
<td></td>
<td>in this area.</td>
</tr>
<tr>
<td></td>
<td>• Children need to see and use a variety of question types during this work</td>
</tr>
</tbody>
</table>
The equals sign is not always correctly interpreted as ‘has the same value as’ by children, who may see it as ‘the answer is’. Including: oral questions ‘thirty-four add forty-eight’, written questions using symbols ‘71 – 34’, simple ‘real’ problems such as shopping and abstract problems such as finding as many pairs of numbers with a sum of 41 as possible and spot the pattern.

- Try to model the wide range of language used to signify addition and subtraction – see vocabulary list above. The children ultimately need to be able to recognise that a problem is an addition problem from the language (and same for subtraction).
- Use ‘sum’ only to mean an addition calculation – use the word ‘calculations’ to mean mixed operation computations.
- Challenge issues with the use of the = sign by looking at examples where the question is on the right e.g. ? = 4 + 8 as well as balance problems in Further Extension e.g. 3 + 4 = ? + 2

<table>
<thead>
<tr>
<th>Key Assessment Checklist</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. I can quickly recall addition and subtraction facts to 20 and use them to derive facts to 100</td>
</tr>
<tr>
<td>2. I can mentally add and subtract ones to and from a two-digit number</td>
</tr>
<tr>
<td>3. I can add and subtract tens to and from a two-digit number</td>
</tr>
<tr>
<td>4. I can add and subtract two two-digit numbers that do not bridge ten with equipment.</td>
</tr>
<tr>
<td>5. <strong>I can add any two two-digit numbers (using apparatus and recording)</strong></td>
</tr>
<tr>
<td>6. <strong>I can subtract any two two-digit numbers (using apparatus and recording)</strong></td>
</tr>
<tr>
<td>7. I can add three one digit numbers</td>
</tr>
<tr>
<td>8. I can say whether the order matters in a statement</td>
</tr>
</tbody>
</table>
### Stage 2

#### Unit 4: Generalising Arithmetic

This unit is focused on developing fluency in the manipulation of number. At primary level this is focused on arithmetic itself and the methods for four operations particularly; however, this is naturally generalised to thinking about rules of arithmetic more widely at secondary level i.e. algebra. These aspects have been paired together intentionally to help teachers describe algebra as simply a generalisation of number. It is expected that teachers will go back to arithmetic to help students see where the ‘rules’ of algebra come from. Note that the greyed out content is covered elsewhere and hence is not required content here.

<table>
<thead>
<tr>
<th>Prior Learning</th>
<th>Core Learning</th>
<th>Learning Leads to...</th>
</tr>
</thead>
</table>
| ➢ solve one-step problems that involve addition and subtraction, using concrete objects and pictorial representations, and missing number problems such as \( 7 = x - 9 \)  
➢ read, write and interpret mathematical statements involving addition (+), subtraction (−) and equals (=) signs | ➢ Solve problems with addition and subtraction:  
• using concrete objects and pictorial representations, including those involving numbers, quantities and measures  
• applying their increasing knowledge of mental and written methods  
➢ recognise and use the inverse relationship between addition and subtraction and use this to check calculations and solve missing number problems. | ➢ solve problems, including missing number problems, using number facts, place value, and more complex addition and subtraction |

#### Working at expected standard

- recognise the inverse relationships between addition and subtraction and use this to check calculations and work out missing number problems (e.g. \( \Delta - 14 = 28 \)).
- use estimation to check that their answers to a calculation are reasonable (e.g. knowing that 48 + 35 will be less than 100)

#### Working at greater depth:

- solve more complex missing number problems (e.g. \( 14 + \Delta = 15 + 27 \)).

#### Exemplification

1. a) There are 24 children in Class A and 28 children in class B. How many children is this altogether?  
b) There are 53 people on a bus. Some people get off at the bus stop. There are now only 29 people left. How many people got off at the bus stop?

2. a) Joey thinks of number. He adds 14 and the answer is 41. What is Joey’s number?  
b) Amy has some string. She cuts off a piece that is 23cm long. Amy is left with only 55cm of string. Amy says “I must have started with 22cm string”. Do you agree with Amy?

#### Vocabulary

- add  
- addition  
- plus  
- more  
- increase  
- altogether  
- total  
- sum  
- subtract  
- subtraction  
- take away  
- leaves / left  
- represent  
- apparatus  
- tens and ones  
- number line  
- bar model  
- part-part-whole  
- inverse  
- opposite  
- fact family  
- commutative  
- missing number  
- solve
<table>
<thead>
<tr>
<th>Representation</th>
<th>Fluency</th>
<th>Probing Questions</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Addition</strong></td>
<td>1. Add two two-digit numbers together (recap)</td>
<td>Show me how you can add 34 and 47 together using equipment</td>
</tr>
<tr>
<td></td>
<td>- two-digit add one-digit</td>
<td>Convince me that 39 + 45 = 84</td>
</tr>
<tr>
<td></td>
<td>- two-digit add two-digit (no exchange)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>- two-digit add two-digit (exchanging from 1s to 10s)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>2. Recognise, represent and solve a simple addition problem (2-digit numbers)</td>
<td>Show me two numbers that sum to 83</td>
</tr>
<tr>
<td></td>
<td>- simple one-step word problem with addition trigger word and given structure for representation (e.g. blank bar model or blank part-part-whole)</td>
<td>Show me three numbers that sum to 100</td>
</tr>
<tr>
<td></td>
<td>- simple one-step word problem with addition trigger word (without scaffolded structure)</td>
<td>Always, Sometimes, Never?</td>
</tr>
<tr>
<td></td>
<td>- one-step addition word problem with subtle reference to addition</td>
<td>Addition is commutative i.e. the order doesn’t matter</td>
</tr>
<tr>
<td></td>
<td>- two-step problems requiring two additions</td>
<td></td>
</tr>
<tr>
<td><strong>Representing Addition Problems</strong></td>
<td>3. Subtract a two-digit number from a two-digit number (recap)</td>
<td>Show me how we can find 76 - 48 using equipment</td>
</tr>
<tr>
<td></td>
<td>- two-digit subtract one-digit</td>
<td>Always, Sometimes, Never?</td>
</tr>
<tr>
<td></td>
<td>- two-digit subtract two-digit (no exchange)</td>
<td>Subtraction is commutative i.e. the order doesn’t matter</td>
</tr>
<tr>
<td></td>
<td>- two-digit subtract two-digit (exchanging from 10s to 1s)</td>
<td></td>
</tr>
<tr>
<td><strong>Subtraction</strong></td>
<td>1. <strong>Representing</strong> first number using tens and ones equipment then removing or taking away the second number and finding the resulting value (partitioning) (exchanging one 10 for ten 1s as required when bridging)</td>
<td></td>
</tr>
<tr>
<td></td>
<td><strong>Progression of</strong> tens and ones equipment:</td>
<td></td>
</tr>
<tr>
<td></td>
<td>- separable: bundles of straws or sticks of multilink cubes</td>
<td></td>
</tr>
<tr>
<td></td>
<td>- inseparable: Dienes rods or Base 10 or Numicon 10s and 1s</td>
<td></td>
</tr>
<tr>
<td></td>
<td>- not to scale: place value counters or money</td>
<td></td>
</tr>
<tr>
<td></td>
<td>- Representing addition as counting or jumping on (augmentation) using:</td>
<td></td>
</tr>
<tr>
<td></td>
<td>- a number track</td>
<td></td>
</tr>
<tr>
<td></td>
<td>- a bead string (reading the answer using 10s and 1s)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>- a hundred square (just a number track split into rows – jumping in 10s and 1s)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>- a marked number line (jumping in 10s and 1s)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>- an unmarked number line</td>
<td></td>
</tr>
</tbody>
</table>
### Representing Subtraction Problems:

- **Representing subtraction word problems using the bar model or a part-part-whole model**

#### 2-Step Problems

- Representing problems using the bar model
  
  **Example:**
  
  There are 37 people on a bus. 17 people get on the bus at the next stop.

#### Representing subtraction as counting or jumping back (reduction) using:

- a number track
- a hundred square (just a number track split into rows – jumping in 10s and 1s)
- a marked number line (jumping in 10s and 1s)
- an unmarked number line

#### Representing subtraction as a comparative difference between two sets of objects using:

- counters
- beadstrings
- Numicon tens and ones (and other tens and ones equipment) laid out in two lines
- number lines with both numbers marked and difference found

### Recognise, represent and solve

- **Recognise, represent and solve a simple subtraction problem (2-digit numbers)**
  
  - simple one-step word problem with subtraction trigger word and given structure for representation (e.g. blank bar model or blank part-part-whole)
  
  - simple one-step word problem with subtraction trigger word (without scaffolded structure)
  
  - one-step subtraction word problem with subtle reference to subtraction
  
  - two-step problems requiring two subtractions

- **Recognise, represent and solve two-step problems combining addition and subtraction**
  
  - simple problems where operation is clear (e.g. people getting on and off a bus or items going in and out of a bag) and structure for addition and subtraction to solve this problem 'There were 19 people on the train then 14 got off and 21 got on. How many are left?'

---

**Show me two numbers that have a difference of 27**

**Convince me that if I have 38 pencils and I take away 13, there are 25 left.**

**Convince me that you need to use addition and subtraction to solve this problem 'There were 19 people on the train then 14 got off and 21 got on. How many are left?''
<table>
<thead>
<tr>
<th>Checking Calculations</th>
<th>Missing Number Problems</th>
<th>What’s the same and what’s different?</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Reversing a calculation to check an answer by adding or subtracting to see if this gives the original value</td>
<td>• Representing using a bar model</td>
<td>Addition and Subtraction</td>
</tr>
</tbody>
</table>
| • Rounding numbers to nearest 10 to estimate the value of the calculation first | Example 1:  
\[ \square + 39 = 65 \] | |
| | Example 2:  
\[ 61 = 83 - \square \] | |
| | **What’s the same and what’s different?** | |
| | **Addition and Subtraction** | |
| | **What’s the same and what’s different?** | |
| | **Always, Sometimes, Never?** | |
| | **You check an addition using a subtraction and a subtraction using an addition.** | |
| | **Always, Sometimes, Never?** | |
| | **If I know one number fact, I automatically know 3 others** | |
| | **Always, Sometimes, Never?** | |
| | **You find a missing number in a subtraction by adding.** | |
| | **Always, Sometimes, Never?** | |
| | **If I know that \( ? + \square = 50 \), then 50 – \square = ?** | |
| | **Always, Sometimes, Never?** | |
sweets. Her aunt gives her 18 more sweets, so now she has 42. How many did she start with?

- subtraction problems (smaller amount missing) e.g. Ella bakes 40 cakes. She sells some to her neighbours. She is left with 11 cakes. How many did Ella sell?
- subtraction problems (larger amount missing) e.g. Some football teams enter a competition. 14 football teams drop out, so there are now only 52 football teams left. How many football teams entered at first?
- mixture of addition/subtraction problems where children select appropriate operation
- 2-step problems e.g. There are some people on a bus. 17 more people get on. Then 24 people get off. There are now 30 people on the bus. How many people were there to start with?

### Combination Missing Number Problems

- Representing balancing problems e.g. \( 15 + 46 = \_ + 34 \) using scales, real or virtual (could use Numicon as weighted)
- Using bar models to represent these problems

<table>
<thead>
<tr>
<th>15</th>
<th>46</th>
</tr>
</thead>
<tbody>
<tr>
<td>?</td>
<td>?</td>
</tr>
</tbody>
</table>

10. (Ext) Solve combination missing number problems

| 23 + 41 = 72 − □ |
| 72 − □ = 23 + 41 |
| 23 + 41 = □ − 72 |

What’s the same and what’s different?

- abstract, all addition e.g. \( 23 + 41 = 72 − □ \)
- abstract, all subtraction e.g. \( 71 − □ = 52 − 39 \)
- abstract, one of each operation e.g. \( 34 + □ = 19 + 51 \)
- worded problems (and hence requiring representation) e.g. Jane is given 15 sweets by her mum and 32 sweets by her uncle. Jane now has the same number of sweets as Paul, who started with 65 but gave some to his sister. How many sweets did Paul give to his friend?
- problems where only a digit of a number is missing e.g. \( 52 − □ 3 = 29 \)
<table>
<thead>
<tr>
<th>Further Extension</th>
<th>Rich and Sophisticated Tasks</th>
</tr>
</thead>
<tbody>
<tr>
<td>Solve problems with addition and subtraction:</td>
<td></td>
</tr>
<tr>
<td>- using concrete objects and pictorial representations, including those involving numbers, quantities and measures</td>
<td>NRICH: Getting the Balance *** I</td>
</tr>
<tr>
<td>- applying their increasing knowledge of mental and written methods</td>
<td>NRICH: Noah ** P</td>
</tr>
<tr>
<td>NRICH: Getting the Balance *** I</td>
<td>NRICH: Eggs in Baskets ** P</td>
</tr>
<tr>
<td>NRICH: The Brown Family *** G P</td>
<td>NRICH: Birthday Cakes ** P</td>
</tr>
<tr>
<td>NRICH: Sitting Round the Party Tables * P I</td>
<td>NRICH: Cuisenaire Counting *** G P</td>
</tr>
<tr>
<td>NRICH: Cuisenaire Counting *** G P</td>
<td>NRICH: Two Spinners * I</td>
</tr>
<tr>
<td>NRICH: Heads and Feet ** P</td>
<td>NRICH: Double or Halve? * G</td>
</tr>
<tr>
<td>NRICH: Double or Halve? * G</td>
<td></td>
</tr>
<tr>
<td>Recognise and use the inverse relationship between addition and subtraction and use this to check calculations and missing number problems</td>
<td>NRICH: The Add and Take-away Path * I</td>
</tr>
<tr>
<td>NRICH: Doing and Undoing * I</td>
<td></td>
</tr>
</tbody>
</table>

1. Using the bar model complete the four number sentences.

Using the bar model complete the four number sentences.

```
[ ] + [ ] = [ ]
[ ] + [ ] = [ ]
[ ] - [ ] = [ ]
[ ] - [ ] = [ ]
```

2. Dan needs 80g of sugar for his recipe. There are 45g left in the bag. How much more does he need to get?

The temperature was 26 degrees in the morning and 11 degrees colder in the evening. What was the temperature in the evening?

A tub contains 24 coins. Saj takes 5 coins. Joss takes 10 coins. How many coins are left in the tub?

3. Together Jack and Sam have £12. Jack has £2 more than Sam. How much money does Sam have?

A bar model can be very helpful in solving these types of problems.

```
Jack +£2
Sam
£12
```

\[ £12 - £2 = £10 \]
\[ £10 ÷ 2 = £5 \]
Sam has £5
<table>
<thead>
<tr>
<th>Misconceptions</th>
<th>Teacher Guidance and Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Children do not use place value when adding and subtracting - signs of this can be them counting repeatedly from 0 or failing to use models that group tens differently. They may need to use a wider range of representations to develop this idea more strongly - some children can do this when the model 'looks' like 10 but not when the visual link has gone.</td>
<td>• In this unit pupils build on their skills of addition and subtraction as developed in Unit 3 to apply them to problems and wider contexts. They also explore the inverse of addition and subtraction, using this to check calculations and to solve missing number problems</td>
</tr>
<tr>
<td>Children confuse the order of their facts e.g. they know that 16 + 5 = 21 and so they say that 21 + 5 = 16. This is because children do not always understand that addition can be done in any order but that subtraction is not commutative.</td>
<td>• However, this unit is also an opportunity to consolidate the skills and fluency of addition and subtraction (particularly if children did not easily master these operations with larger numbers in Unit 3).</td>
</tr>
<tr>
<td>A significant challenge for children in this unit is making the decision about whether to add or subtract the numbers that they have been given. This relies on a solid language understanding of the many variants of words used to imply add and subtract, which may be weak.</td>
<td>• The use of manipulatives and visual representations is stipulated in the objectives of this unit and so these should be available throughout – the representation section gives more guidance on what could/should be used and how</td>
</tr>
<tr>
<td>• It is critical that children learn how to turn a word problem into a number problem and make the decision about whether it is asking them to add or subtract (or possibly both if it is a 2-step problem).</td>
<td></td>
</tr>
<tr>
<td>• The bar model and part-part-whole models are strongly recommended as ways to represent a word problem to help decide which calculation to use.</td>
<td></td>
</tr>
<tr>
<td>• Language here is key – try to model different terms that imply add/subtract to broaden children’s vocabulary</td>
<td></td>
</tr>
<tr>
<td>• Try to avoid implying that it is not possible to subtract a larger number from a smaller number as this may cause problems when introducing negative numbers later on.</td>
<td></td>
</tr>
</tbody>
</table>

### Key Assessment Checklist

1. I can add a two-digit number to a two-digit number using manipulatives, pictures and my head
2. I can subtract a two-digit number from another two-digit number using manipulatives, pictures and my head
3. I can represent and solve addition word problems
4. I can represent and solve subtraction word problems
5. I can check a calculation using the inverse
6. I can use the inverse of addition and subtraction to find missing numbers.
### Stage 2

#### Unit 5: Exploring Shape

8 learning hours

In this unit children and students explore the properties of shapes, both 2D and 3D.

At KS1 this is focused on common shape names and basic features of vertices, sides etc. but this then develops to classifying quadrilaterals and triangles in KS2. Alongside this focus children begin to explore angle and turn in KS2 and develop this to more formal angle rules through Stages 5, 6, 7, 8.

Older students begin to explore the field of trigonometry, encountering first Pythagoras’ Theorem, then RA-triangle trig before finally looking at a sine rule and cosine rule.

<table>
<thead>
<tr>
<th>Prior Learning</th>
<th>Core Learning</th>
<th>Learning Leads to...</th>
</tr>
</thead>
<tbody>
<tr>
<td>➢ recognise and name common 2-D and 3-D shapes, including: - 2-D shapes [for example, rectangles (including squares), circles and triangles] - 3-D shapes [for example, cuboids (including cubes), pyramids and spheres]</td>
<td>➢ identify and describe the properties of 2-D shapes, including the number of sides and line symmetry in a vertical line ➢ identify and describe the properties of 3-D shapes, including the number of edges, vertices and faces ➢ compare and sort common 2-D and 3-D shapes and everyday objects</td>
<td>➢ identify horizontal and vertical lines and pairs of perpendicular and parallel lines ➢ recognise angles as a property of shape or a description of a turn ➢ identify right angles, recognise that two right angles make a half-turn, three make three quarters of a turn and four a complete turn; identify whether angles are greater than or less than a right angle</td>
</tr>
</tbody>
</table>

### Prior Learning

- recognise and name common 2-D and 3-D shapes, including:
  - 2-D shapes [for example, rectangles (including squares), circles and triangles]
  - 3-D shapes [for example, cuboids (including cubes), pyramids and spheres]

### Core Learning

- identify and describe the properties of 2-D shapes, including the number of sides and line symmetry in a vertical line
- identify and describe the properties of 3-D shapes, including the number of edges, vertices and faces
- compare and sort common 2-D and 3-D shapes and everyday objects

### Working towards

- recognise and name triangles, rectangles, squares, circles, cuboids, cubes, pyramids and spheres from a group of shapes or from pictures of the shapes

### Working at expected standard

- describe the properties of 2-D and 3-D shapes (e.g. the pupil describes a triangle: it has 3 sides, 3 vertices and 1 line of symmetry; the pupil describes a pyramid: it has 8 edges, 5 faces, 4 of which are triangles and one is a square)

### Working at greater depth

- describe similarities and differences of shape properties (e.g. finds 2 different 2-D shapes that only have one line of symmetry; that a cube and a cuboid have the same number of edges, faces and vertices but can describe what is different about them).

### Exemplification

1. a) Complete this statement: A hexagon has ........... edges and ........ vertices
   b) I am thinking of a 2D shape. It has 4 sides. What could my shape be? Give two possible answers.
   c) Draw a triangle without a line of symmetry

2. a) Complete this statement: A cuboid has ........ faces, and ........ vertices
   b) I am thinking of a 3D shape. It has a circular base. What could my shape be? Give two possible answers.
   c) True of False: a pyramid has 5 faces

### Vocabulary

<table>
<thead>
<tr>
<th>2D</th>
<th>3D</th>
</tr>
</thead>
<tbody>
<tr>
<td>circle</td>
<td>cube</td>
</tr>
<tr>
<td>triangle</td>
<td>cuboid</td>
</tr>
<tr>
<td>square</td>
<td>sphere</td>
</tr>
<tr>
<td>rectangle</td>
<td>pyramid</td>
</tr>
<tr>
<td>parallelogram</td>
<td>...-based (pyramid)</td>
</tr>
<tr>
<td>rhombus</td>
<td>prism</td>
</tr>
<tr>
<td>quadrilateral</td>
<td>(........ ) prism</td>
</tr>
<tr>
<td>pentagon</td>
<td>cylinder</td>
</tr>
</tbody>
</table>
3. Look at these shapes:
   - Rectangle
   - Pentagon
   - Cone
   - Triangular Prism
   - Square
   - Triangle
   - Circle
   - Cuboid

   Which of these shapes
   a) are quadrilaterals?
   b) have only straight sides/edges?
   c) are prisms?

<table>
<thead>
<tr>
<th>Representation</th>
<th>Fluency</th>
<th>Probing Questions</th>
</tr>
</thead>
<tbody>
<tr>
<td>2D Shapes: Sides and Vertices</td>
<td>1. Describe the sides and vertices of 2D shapes (from diagrams or names)</td>
<td>Show me a shape with three vertices</td>
</tr>
<tr>
<td></td>
<td>• sketch a named 2D shape</td>
<td>Show me a shape with four sides</td>
</tr>
<tr>
<td></td>
<td>• state the number of vertices of a 2D shape</td>
<td>Show me a pentagon</td>
</tr>
<tr>
<td></td>
<td>• state the number of sides of a 2D shape</td>
<td>Convince me that this is a quadrilateral</td>
</tr>
<tr>
<td></td>
<td>• use number of sides/vertices to identify polygons i.e. to say if a shape is a quadrilateral or not or is a pentagon or not</td>
<td>Always, Sometimes, Never?</td>
</tr>
<tr>
<td></td>
<td>• identify whether the sides are straight or curved</td>
<td>Shapes have the same number of sides as vertices</td>
</tr>
<tr>
<td></td>
<td>• identify whether any of the sides are the same length</td>
<td></td>
</tr>
<tr>
<td></td>
<td>• given a description of the sides/vertices of a shape, suggest its name or sketch the shape</td>
<td></td>
</tr>
<tr>
<td>Symmetry</td>
<td>2. Describe the symmetry of 2D shapes</td>
<td></td>
</tr>
<tr>
<td></td>
<td>• identify lines of symmetry on drawn shapes (vertical)</td>
<td>Show me a shape with a line of symmetry</td>
</tr>
<tr>
<td></td>
<td>• identify lines of symmetry on drawn shapes (non-vertical)</td>
<td>Show me a shape with more than 1 line of symmetry</td>
</tr>
<tr>
<td></td>
<td>• identify all the lines of symmetry on a drawn shape</td>
<td>Convince me that this shape has a line of symmetry</td>
</tr>
<tr>
<td></td>
<td>• identify shapes that do not have any lines of symmetry</td>
<td>Convince me that this shape does not have any lines of symmetry</td>
</tr>
<tr>
<td></td>
<td>• say whether a line dividing the shape into two is a line of symmetry or not</td>
<td>Always, Sometimes, Never?</td>
</tr>
<tr>
<td></td>
<td>• complete a shape given half of it and a vertical mirror line</td>
<td>2D shapes have 1 line of symmetry</td>
</tr>
<tr>
<td>Properties of 2D Shapes</td>
<td>3. Compare 2D shapes</td>
<td>4. Identify/draw (sketch) a 2D shape given its properties</td>
</tr>
<tr>
<td>------------------------</td>
<td>---------------------</td>
<td>---------------------------------</td>
</tr>
</tbody>
</table>
| • Folding paper in half in different ways. If they make one fold on a square, what shapes can they make? Can they make one fold and make a four-sided shape that isn’t a rectangle?  
• Playing with tangrams to explore the properties of a square. **Nrich** version  
• Using shapes to make patterns and describing these using mathematical language  
• Using geoboards and elastic bands to make shapes  
• Making shapes out of identical triangle pieces and naming these  
• Using the **Polygon ITP** to explore shapes with ICT  
• Using plastic geostrips to investigate triangles or quadrilaterals with sides of different lengths | • by number of sides (including ordering 3 or more)  
• by number of vertices (including ordering 3 or more)  
• by type of sides e.g. curved or straight  
• by length of sides i.e. sides that are equal  
• by lines of symmetry  
• by whether they meet a definition e.g. quadrilateral or triangle | • given a property, suggest a possible shape e.g. four sides or all sides are equal length  
• given a property, suggest all possible shapes e.g. four sides or all sides are equal length  
• given several properties, pinpoint the exact shape e.g. three sides, one line of symmetry |  
| 4. Identify/draw (sketch) a 2D shape given its properties | Show me a shape where all the sides are equal length  
Convince me that this is a rectangle  
What’s the same and what’s different? triangle, rectangle, pentagon, circle  
Always, Sometimes, Never? A square is a type of rectangle | Show me all the shapes with no vertices  
Show me all the shapes with a right angle |
### 3D Shapes

- Exploring and handling 3D shapes using mathematical models as well as everyday objects, especially packaging, to help count the faces, edges and vertices.
- Going on a shape hunt to find specific shapes in real life e.g. cuboids.
- Printing with 3D shapes to explore the shapes of the faces. Which shapes have square faces? Did you print with any shapes with circular faces? What happens when you print with a sphere? A cylinder?
- Pulling 3D shapes apart (e.g. packets) to see the 2D shapes that they are made from.
- Making models of shapes using plasticine or using construction materials (e.g. blocks, duplo, multi-link etc).
- Making models using straws and balls of modelling clay or equivalent. Then exploring how many straws and connecting balls you need to make a cube? A pyramid?

### Describing 3D Shapes

5. Describe the faces, edges and vertices of 3D shapes:
- sketch or find a named 3D shape
- state the number of faces of a 3D shape
- identify whether the faces are flat or curved
- identify whether any of the faces are the same
- name the shapes of each face of a 3D shape (not curved faces)
- state/count the number of vertices of a 3D shape
- state/count the number of edges of a 3D shape
- given a description of the faces/vertices/edges of a shape, suggest its name

| Show me a sphere |
| Show me a shape with six faces |
| Show me all the shapes that are 3D |
| Convince me that this shape will roll |
| Always, Sometimes, Never? |
| A pyramid has one square face |

### Identifying 3D Shapes

- Playing peekaboo with shapes, trying to guess the shape as more and more is revealed.
- Playing ‘Guess My Shape’ using 20 questions format (can be done nicely using a shape fan so that all children put forward a guess after each new fact is revealed).

### Comparing 3D Shapes

6. Compare 3D shapes:
- by number of faces
- by shapes of faces
- by number of vertices
- by whether faces are curved or straight
- by whether they meet a category definition e.g. pyramid or prism

| Convince me that a cube and a cuboid have the same number of faces, edges and vertices |
| Convince me that a cuboid is a prism |

### Sorting Shapes

- Finding all the shapes that match a given rule e.g. a shape with 6 vertices or a shape with all sides the same length.
- Organising a set of shapes into groups of the child’s own choosing and explaining how they have organised them.
- Making a shape family tree.
- Finding all the shapes that match a given criteria.

### Identifying 2D and/or 3D Shapes using given criteria

8. Sort and classify 2D and/or 3D shapes using given criteria:
- sort sets of shapes into 2 groups e.g. 2D/3D or no curved edges/has curved edges.
- sort sets of shapes into more complex groups e.g. triangles; quadrilaterals; shapes with 5 more sides; others.
- sort sets of shapes using 2 criteria e.g. 2D/3D AND no curved edges/curved edges.

<p>| What’s the same and what’s different? |
| cube; cuboid; square; sphere |
| What’s the same and what’s different? |
| circle; sphere; cylinder; cube |</p>
<table>
<thead>
<tr>
<th>Further Extension</th>
<th>Rich and Sophisticated Tasks</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Cut a square piece of paper as shown. Rearrange the pieces to make different shapes. What different shapes can you make?</td>
<td>Identify and describe the properties of 2-D shapes, including the number of sides and line symmetry in a vertical line</td>
</tr>
<tr>
<td>Describe the properties of the shapes you make. Can you make some shapes which have at least one line of symmetry?</td>
<td>NRICH: Shapely Lines * I</td>
</tr>
<tr>
<td>2. Captain Conjecture says, 'All of these shapes are rectangles because they have four sides.' Do you agree?</td>
<td>NRICH: Chain of Changes ** P</td>
</tr>
<tr>
<td>Explain your reasoning.</td>
<td>NRICH: Colouring Triangles ** P I</td>
</tr>
<tr>
<td>3. Jack has made a cube using 12 sticks and 8 balls of modelling clay.</td>
<td>NRICH: Exploded Squares * P</td>
</tr>
<tr>
<td>What shape could he make with: 6 sticks and 4 balls of clay? 4 long sticks, 8 short sticks 8 balls of clay?</td>
<td>NRICH: Complete the Square *** G</td>
</tr>
<tr>
<td></td>
<td>NRICH: Let's Investigate Triangles * P</td>
</tr>
<tr>
<td></td>
<td>NRICH: Poly Plug Rectangles * G I</td>
</tr>
<tr>
<td></td>
<td>NRICH: Square It * G</td>
</tr>
<tr>
<td></td>
<td>NRICH: Inside Triangles *** G P</td>
</tr>
<tr>
<td></td>
<td>Identify and describe the properties of 3-D shapes, including the number of edges, vertices and faces</td>
</tr>
<tr>
<td></td>
<td>NRICH: Building with Solid Shapes * I</td>
</tr>
<tr>
<td></td>
<td>NRICH: Skeleton Shapes ** P I</td>
</tr>
<tr>
<td></td>
<td>NRICH: Rolling That Cube * I</td>
</tr>
<tr>
<td>Compare and sort common 2-D and 3-D shapes and everyday objects</td>
<td>Other</td>
</tr>
<tr>
<td>NRICH: Matching Triangles * G</td>
<td>NRICH: Take a .... Geoboard</td>
</tr>
<tr>
<td>NRICH: Data Shapes * P</td>
<td>NRICH: Properties of Shapes KS1</td>
</tr>
<tr>
<td>NRICH: Cubes Cut into Four Pieces *** P</td>
<td>NRICH: Stringy Quads</td>
</tr>
<tr>
<td>NRICH: Let us reflect</td>
<td>NRICH: Let us reflect</td>
</tr>
</tbody>
</table>
### Misconceptions

As in Stage 1, a key misconception occurs where children relate the orientation of a shape to its definition. For example, they fail to recognise a square that has been rotated or a cone whose circular face is not at the base of the page/on the surface. In fact, children can infer that part of the definition of a shape is its colour etc. if this is how they always see that particular shape presented so ensure these presentations are varied to avoid inadvertently implying a property that is not intended!

Some children believe that the terms corner and vertex are not interchangeable - however, they are (and apply to both 2D and 3D shapes). Distinguishing between these can help separate 2D and 3D shapes, however.

Children miscount vertices and sides sometimes because they forget where they have started counting.

Some children are surprised that there is more than one shape with, for example, four sides and they therefore assume that it must be a square because they know a square has four sides.

Relatedly, they do not recognise ‘quadrilateral’ as a category of shape that other shapes fit into; instead they see it a distinct shape.

In a similar way, the inclusive definition of rectangle, for example, which includes squares, is not always clear to children who believe they are distinct.

Children can sometimes identify lines of symmetry that are not present because the area and shape of the two pieces they create are the same, although they are not correctly oriented to give reflection symmetry. E.g. they believe a rectangle has a (two) diagonal line(s) of symmetry because it creates two seemingly identical shapes when you cut it down the diagonal. Children find symmetry lines particularly hard to spot when they are not vertical to the page.

### Teacher Guidance and Notes

- In this Stage we are trying to develop children’s use of mathematical language to describe shapes and their properties more technically.
- There is no specific range of shapes that should be covered, however it is recommended that those highlighted in the vocabulary box be used.
- You need to ensure that children are beginning to be able to use category words for shapes also, such as quadrilateral or pyramid, and that they recognise that these names cover multiple shapes.
- Be aware that most of the standard sets of shapes that you can buy or print represent larger polygons in their regular form. Try to use some examples of irregular pentagons, hexagons etc so that children do not assume that all pentagons have equal length sides and 5 lines of symmetry for example. This belief may stop them recognising a shape as a pentagon, even when it is!
- Don’t be frightened of introducing a wider range of shapes - many children of this age can handle a significant amount of new vocabulary such as hemisphere or cylinder or octagon. You may even begin to be able to look at types of triangle with the most able to see the difference between an equilateral and the others. You may NEED to do this to show the children some shapes WITHOUT symmetry .....
- Consider making each child a shape fan (like a number fan!) that they can use for show-me activities on the carpet or for investigating shape features.
- Be aware that we are ultimately moving to the idea that a rectangle is the family name and the family divides into squares and oblongs - if you can prepare the way for this it will help!
- When trying to teach children the meaning of a property e.g. has a line of symmetry or is a triangle, it can be very helpful to spend time looking at non-examples to show why they do not meet the criterion. This helps children to begin to understand what qualifies and what does not. For example, ask “why is this not a quadrilateral/?prism/?3D shape/?symmetrical shape?”
**Key Assessment Checklist**

1. I can recognise and name 2D and 3D shapes
2. I can identify the number of sides and vertices of a 2D shape
3. I can recognise if a 2D shape has a vertical line of symmetry and say/show where this is
4. I can identify the number of faces, edges and vertices on a 3D shape.
5. I can describe the faces of a 3D shape
6. I can compare 2D shapes
7. I can compare 3D shapes
8. I can sort 2D and 3D shapes into groups using given criteria.
### Stage 2

#### Unit 6: Reasoning with Measures

**8 learning hours**

This unit focuses on mensuration and particularly the concepts of perimeter, area, and volume. Primary children are also working on money concepts at this stage, while older secondary students develop mensuration into volume and surface area of challenging shapes, applying Pythagoras' Theorem and trigonometry also in combination with these problems.

Note the focus on reasoning within this unit: it is common for children to complete routine problems involving mensuration but this unit is about the developing a secure conceptual understanding of these ideas that they can apply to a wide range of problems and contexts. The opportunity to use and build on earlier number work is built into this unit and it is expected that children apply their arithmetic skills, for example, in these problems.

#### Prior Learning

- Recognise and know the value of different denominations of coins and notes

#### Core Learning

- Recognise and use symbols for pounds (£) and pence (p); combine amounts to make a particular value
- Find different combinations of coins that equal the same amounts of money
- Solve simple problems in a practical context involving addition and subtraction of money of the same unit, including giving change

*Working at expected standard*

- Use different coins to make the same amount (e.g. pupil uses coins to make 50p in different ways; pupil can work out how many £2 coins are needed to exchange for a £20 note)

#### Exemplification

1. a) Write these amounts using numbers and the £ and p signs
   - (i) Four pounds
   - (ii) Sixty-four pence
   - (iii) Two pounds and thirty pence
   b) Write 150p using the £ symbol

2. a) Find three coins that make 60p.
   b) Find a different set of three coins that make 60p.

3. Shaheen buys an apple for 15p and a banana for 26p. She pays using a 50p coin. Find the right coins for Shaheen’s change.

#### Vocabulary

- Money
- Coin
- Note
- Pound
- £
- Pence
- P
- Total
- Amount
- Price
- Cost
- Altogether
- Pay
- Shop
- Bill
- Change
- Subtract
- Less
- Take away
- Difference
### Representation

<table>
<thead>
<tr>
<th>Money</th>
<th>Fluency</th>
<th>Probing Questions</th>
</tr>
</thead>
<tbody>
<tr>
<td>handling coins to learn their values and features e.g. colour, shapes</td>
<td>1. Recognise and use 1p, 2p and 5p coins</td>
<td>Show me all the ways I can make 9p using coins</td>
</tr>
<tr>
<td>making amounts out of given coins</td>
<td>• identify the value given the coin</td>
<td>Convince me that I can make any amount out of 1p, 2p and 5p coins</td>
</tr>
<tr>
<td>finding different coins that give the same total (and trying to get all the possible combinations)</td>
<td>• identify the coin given the value</td>
<td></td>
</tr>
<tr>
<td>finding total of coins and sharing into piggy banks – see this <a href="#">Coin matching computer activity</a> (from Nationwide)</td>
<td>• identify and record the value of a combination of 1ps and 2ps</td>
<td></td>
</tr>
<tr>
<td></td>
<td>• identify and record the value of a combination of 1ps, 2ps and 5ps</td>
<td></td>
</tr>
<tr>
<td></td>
<td>• find coins (1ps, 2ps, 5ps) to make a given amount e.g. 12p</td>
<td></td>
</tr>
<tr>
<td></td>
<td>2. Recognise and use 10p, 20p and 50p coins</td>
<td>Show me where 50p lies on this paper strip from £0 to £1 (£0 to £2 etc)</td>
</tr>
<tr>
<td></td>
<td>• identify the value given the coin</td>
<td>Show me all the ways I can make 24p using coins</td>
</tr>
<tr>
<td></td>
<td>• identify the coin given the value</td>
<td>Convince me that there is only one way to make 20p using 3 coins</td>
</tr>
<tr>
<td></td>
<td>• identify and record the value of a combination of 10ps and 20ps</td>
<td>Convince me that you cannot make 58p out of 3 coins</td>
</tr>
<tr>
<td></td>
<td>• identify and record the value of a combination of 10ps, 20ps and 50ps</td>
<td></td>
</tr>
<tr>
<td></td>
<td>• find coins (10ps, 20ps, 50ps) to make a given amount (&lt;100) e.g. 90p</td>
<td></td>
</tr>
<tr>
<td></td>
<td>• find coins (10ps, 20ps, 50ps) to make a given amount (&gt;100) e.g. 150p</td>
<td></td>
</tr>
<tr>
<td>3. Recognise and use £1 and £2 coins and notes</td>
<td>• identify the value given the coin</td>
<td>Always, Sometimes, Never? Every amount can be made from 1, 2 or 3 coins.</td>
</tr>
<tr>
<td></td>
<td>• identify the coin given the value</td>
<td></td>
</tr>
<tr>
<td></td>
<td>• identify and record the value of a combination of £1s and £2s</td>
<td></td>
</tr>
<tr>
<td></td>
<td>• identify and record the value of a combination of £1s, £2s, £5 notes and £10 notes.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>• find coins (£1/£2) to make a given amount e.g. £9</td>
<td></td>
</tr>
<tr>
<td></td>
<td>• find coins and notes to make a given amount e.g. £9, £23</td>
<td></td>
</tr>
<tr>
<td>4. Use all coins and notes simultaneously</td>
<td>• identify and record the value of a combination of coins</td>
<td>Show me which amounts can be made using only 2 coins</td>
</tr>
<tr>
<td></td>
<td>• identify and record the value of a combination of coins and/or notes</td>
<td></td>
</tr>
<tr>
<td>Understanding Equivalence</td>
<td>Find the total of some items</td>
<td>Shopping and Change</td>
</tr>
<tr>
<td>---------------------------</td>
<td>-----------------------------</td>
<td>---------------------</td>
</tr>
<tr>
<td>- Finding different coins that match £1 to explore the idea of 100 pence = 1 pound</td>
<td>- two items, in whole pounds, no bridging</td>
<td>- Playing shops to practise finding the right coins, working out the total, working out the change and giving correct coins for change</td>
</tr>
<tr>
<td>- Replacing £1 coins with 100 place value counters to help convert between pounds and pence</td>
<td>- two items, in whole pounds, bridging</td>
<td>- Exploring which items can be bought using exact money with coins and items provided. For example (from N:Rich)</td>
</tr>
<tr>
<td>5. Understand the equivalence of £ and pence</td>
<td>- two items, in pence, no bridging (&lt;100)</td>
<td>7. Choose appropriate coins to pay for a purchase</td>
</tr>
<tr>
<td>- know that £1 is the same 100p</td>
<td>- two items, in pence, bridging (&lt;100)</td>
<td>- exact amount</td>
</tr>
<tr>
<td>- say/write a whole amount given in pounds in pence e.g. £2 or £5</td>
<td>- three or more items, in pence (&lt;100)</td>
<td>- estimate amount and give appropriate coin</td>
</tr>
<tr>
<td>- say/write a decimal amount given in pounds in pence e.g. £1.50 or £1.20</td>
<td>- two items, in pence, &gt;100</td>
<td>8. Find the change for a purchase</td>
</tr>
<tr>
<td>- say/write an amount given in pence in pounds e.g. 250p (ext: 85p)</td>
<td>- ext: two items in pounds (not whole number)</td>
<td>- one item, whole pounds e.g. £5-£2</td>
</tr>
<tr>
<td>- ext: say how many pence are equivalent to amounts in pounds &lt;1 e.g. £0.50 or £0.35</td>
<td>- ext: mixed units e.g. £1.50 + 65p</td>
<td>- one item, pence, no bridging e.g. 50p – 30p</td>
</tr>
<tr>
<td>6. Find the total of some items</td>
<td>7. Choose appropriate coins to pay for a purchase</td>
<td>Convince me that a 50p coin will be enough to pay for two items costing 21p and 18p.</td>
</tr>
<tr>
<td>- two items, in whole pounds, no bridging</td>
<td>- one item, whole pounds e.g. £5-£2</td>
<td>Show me how to give change when I buy things for 12p and 5p and give a 20p</td>
</tr>
<tr>
<td>- two items, in whole pounds, bridging</td>
<td>- one item, pence, no bridging e.g. 50p – 30p</td>
<td>What’s the same and what’s different? Change; subtraction; addition; total cost</td>
</tr>
<tr>
<td>- two items, in pence, no bridging (&lt;100)</td>
<td>- one item, pence, bridging e.g. 70p - 59p</td>
<td>Always, Sometimes, Never? The shopkeeper will be able to give the right change.</td>
</tr>
<tr>
<td>- two items, in pence, bridging (&lt;100)</td>
<td>- one item, pence and pounds e.g. £1 – 85p</td>
<td></td>
</tr>
<tr>
<td>- three or more items, in pence (&lt;100)</td>
<td>- two items, all pence</td>
<td></td>
</tr>
<tr>
<td>- two items, in pence, &gt;100</td>
<td>- two items, pence and pounds</td>
<td></td>
</tr>
<tr>
<td>- ext: two items in pounds (not whole number)</td>
<td>given the change and amount paid, say how much the item(s) cost</td>
<td></td>
</tr>
<tr>
<td>- ext: mixed units e.g. £1.50 + 65p</td>
<td>Convince me that 20p is not enough money to buy three 7p sweets</td>
<td></td>
</tr>
</tbody>
</table>

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### Further Extension

1. Holly uses a £1 coin to buy a pack of stickers. Here is the change she was given.

![Change White Elephant](https://crickweb.net/)

How much did the pack of stickers cost?

2. I spend £2 on a drink and sandwich. The sandwich costs 80p more than the drink. How much does the sandwich cost?

3. Grace uses a £1 coin to buy a can of drink which costs 80p. She is given three coins in change. What coins could she have been given?

4. Grace uses a £2 coin to buy a can of drink which costs 85p. She is given four coins in change. Find all the possible combinations of coins she could have been given.

### Rich and Sophisticated Tasks

- Recognise and use the symbols for pounds (£) and pence (p); combine amounts to make a particular value
  - NRICH: [Five Coins](https://nrich.maths.org/)
  - NRICH: [Money Bags](https://nrich.maths.org/)

- Find different combinations of coins that equal the same amounts of money
  - NRICH: [Five Coins](https://nrich.maths.org/)
  - NRICH: [Money Bags](https://nrich.maths.org/)

- Solve simple problems in a practical context involving addition and subtraction of money of the same unit, including giving change
  - NRICH: [Five Coins](https://nrich.maths.org/)
  - NRICH: [Money Bags](https://nrich.maths.org/)
  - NRICH: [The Puzzling Sweet Shop](https://nrich.maths.org/)

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- Practising giving change using coins or using a computer simulation e.g. [Change White Elephant](https://crickweb.net/) from Crickweb

- Always, Sometimes, Never?
  - There is more than one way to give any amount of change.
5. Sid says, ‘I have bought 2 items for my holiday. One item cost £9 more than the other. I spent over £15.’ What two items did Sid buy? The ______ and the ______.

Make up your own problems using the holiday items.

6. Look at these coins. How could you make up the same total amount using just one type of coin?

50p 10p 10p

5p 5p

7. Sam says I can make 97p using just four coins. Is he correct?

Explain your reasoning.

<table>
<thead>
<tr>
<th>Misconceptions</th>
<th>Teacher Guidance and Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Children may find it hard to see</td>
<td>• This unit is focused on developing confidence and fluency with money in real settings.</td>
</tr>
<tr>
<td>that an amount over 100p can be</td>
<td>• Therefore, it is crucial that children have the opportunity to work in context for much</td>
</tr>
<tr>
<td>written in pence or in pounds e.g.</td>
<td>of this unit.</td>
</tr>
<tr>
<td>160p is the same as £1.60</td>
<td>• The pitch of the work is at the level of children’s arithmetic skills i.e. 2-digit</td>
</tr>
<tr>
<td>Some children may write p as 9</td>
<td>amounts in pence and whole numbers of pounds.</td>
</tr>
<tr>
<td>Weak arithmetic skills can cause</td>
<td></td>
</tr>
<tr>
<td>issues in selecting appropriate</td>
<td></td>
</tr>
<tr>
<td>coins and notes.</td>
<td></td>
</tr>
</tbody>
</table>
Sometimes children treat all coins of the same colour as the same value i.e. they see 1p and 2p coins as the same etc.

Some children struggle to grasp the concept of not having right money and needing change.

However, it also expected that children will understand the equivalence of £1 and 100p and use this to find change in very simple decimal situations such as a cost of £1.50 and someone paying £2 or £5. This is achievable due to the focus on using the actual coins, rather than seeing this abstractly at this stage.

- A critical learning point is the ability to record, using correct terminology. Children can show creativity in finding all the ways of making eg 11p (11 x 1p, 2p + 9 x 1p, etc) but will probably record using ‘+’ only (eg 5p + 2p + 2p + 2p = 11p).
- To support this work you can use coins to represent arithmetic problems/calculations to help children cement them as a legitimate model/representation for number.
- The idea of giving change is not always easy for pupils and may require some work. It can link to subtraction on a number line if the ‘jumps’ are restricted to coin amounts.

### Key Assessment Checklist

1. I can make small amounts of money in different ways using 1p and 2p coins and record the result (eg as 7p = 2p + 2p + 2p + 1p)
2. I can make small amounts of money in different ways using 1p, 2p, 5p and 10p coins and record the result (eg as 17p = 10p + 5p + 2p)
3. I can make larger amounts of money in different ways using any coins and record the result using p or £ (eg as £1.50 = £1 + 50p)
4. I can add up and record the total cost of 2 or 3 items in pence up to 99p.
5. I can select appropriate coins to pay for one or more items in a pretend shop
6. I can add up and record the total cost of 2 or 3 items where the total is over £1.
7. I can use counting on or formal subtraction to work out change.
8. I can select appropriate coins to give change for an item in a pretend shop
### Stage 2

#### Unit 7/8: Discovering Equivalence and Reasoning with Fractions

11 learning hours

This unit is a combination of two units that are separated in older year groups to allow teachers extra time to master the concepts.

The unit explores the concepts of fractions (decimals and percentages) as ways of representing non-whole quantities and proportions.

For the youngest children, the work is focused on fractions and developing security in recognising and naming them.

At KS2 this then builds to looking at families of fractions and decimals and percentages.

At secondary level this is extended to more complex percentage work and equivalence with recurring decimals and surds.

<table>
<thead>
<tr>
<th>Prior Learning</th>
<th>Core Learning</th>
<th>Learning Leads to...</th>
</tr>
</thead>
<tbody>
<tr>
<td>➢ recognise, find and name a half as one of two equal parts of an object, shape or quantity</td>
<td>➢ recognise, find, name and write fractions 1/3, 1/4, 2/4 and 3/4 of a length, shape, set of objects or quantity</td>
<td>➢ recognise, find and write fractions of a discrete set of objects: unit fractions and non-unit fractions with small denominators</td>
</tr>
<tr>
<td>➢ recognise, find and name a quarter as one of four equal parts of an object, shape or quantity</td>
<td>➢ write simple fractions for example, 1/2 of 6 = 3 and recognise the equivalence of 2/4 and ½</td>
<td>➢ recognise and use fractions as numbers: unit fractions and non-unit fractions with small denominators</td>
</tr>
<tr>
<td>➢ identify and represent numbers using objects and pictorial representations including the number line, and use the language of: equal to, more than, less than (fewer), most, least</td>
<td>➢ identify, represent and estimate numbers using different representations, including the number line</td>
<td>➢ compare and order unit fractions, and fractions with the same denominators</td>
</tr>
</tbody>
</table>

**Working at expected standard**

- identify \( \frac{1}{3} \), \( \frac{1}{4} \), \( \frac{2}{3} \), \( \frac{2}{4} \), \( \frac{3}{4} \) and knows that all parts must be equal parts of the whole

**Working at greater depth:**

- find and compare fractions of amounts (e.g. 1/4 of £20 = £5 and 1/2 of £8 = £4 so 1/4 of £20 is greater than 1/2 of £8).

---

### Exemplification

**Vocabulary**

<table>
<thead>
<tr>
<th>fraction</th>
<th>half ( \frac{1}{2} )</th>
<th>quarter</th>
</tr>
</thead>
<tbody>
<tr>
<td>one quarter ( \frac{1}{4} )</td>
<td>two quarters ( \frac{2}{4} )</td>
<td>three quarters ( \frac{3}{4} )</td>
</tr>
<tr>
<td>one third ( \frac{1}{3} )</td>
<td>parts</td>
<td>whole</td>
</tr>
<tr>
<td>equal</td>
<td>numerator</td>
<td></td>
</tr>
</tbody>
</table>

1. Name the fractions represented by the black section in each picture:

   a) ![Fraction a](image1)
   b) ![Fraction b](image2)
   c) ![Fraction c](image3)
   d) ![Fraction d](image4)
2. a) I have 10 counters. Half of them are red. How many are red?  
b) \(\frac{1}{4}\) of 8 is .......

3. Represent the fraction \(\frac{1}{3}\) in 3 different ways. You can use these diagrams to help you if you wish.

<table>
<thead>
<tr>
<th>Representation</th>
<th>Fluency</th>
<th>Probing Questions</th>
</tr>
</thead>
</table>
| **Recognising and Writing Fractions** | 1. Recognise and write fractions of shapes/sets of objects  
- state the number of parts in the whole (of a shape/set of objects)  
- know that these parts must be of equal size (and say when they are not)  
- state the number of parts that are shaded/have a specific property  
- recognise and say the fraction of a shape that has been shaded when all equal parts are of the same shape (half, a quarter, a third)  
- recognise and say the fraction of a shape that has been shaded when all equal parts are of the same shape (two quarters, three quarters)  
- recognise and say the fraction of a shape that has been shaded when the equal parts are of different shapes (half, a quarter, a third, two quarters, three quarters)  
- write the fraction of a shape that has been shaded \(\frac{1}{2}, \frac{1}{4}, \frac{2}{4}, \frac{3}{4}, \frac{1}{3}\)  
- recognise and say the fraction of a set of objects (2, 3 or 4) that have a given property e.g. fraction of these four cars that are red  
- write this as a fraction \(\frac{1}{2}, \frac{1}{4}, \frac{2}{4}, \frac{3}{4}, \frac{1}{3}\) | Convince me that \(\frac{1}{2}\) of this shape has been shaded  
What’s the same and what’s different?  
1/4, 2/4, 3/4, 4/4  
What’s the same and what’s different?  
Part and Whole |
| **Representing Half as Proportions of a Shape/Object** | 2. Find and recognise half of a shape  
- find half of a shape with two marked equal parts of same shape  
- find half of a shape by splitting it into two equal parts (in different ways if appropriate)  
- find half of a shape with four marked equal parts | Show me \(\frac{1}{2}\) of these shapes |
| symmetrical paper shapes in half to find half of the shape | • find half of a shape with six, eight or ten marked equal parts  
• find half of a shape with marked equal parts of different shapes |
<table>
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</thead>
<tbody>
<tr>
<td>• Specifically folding (and colouring) paper strips to show ( \frac{1}{2} )</td>
<td>e.g.</td>
</tr>
<tr>
<td>• Finding different ways to fold a post-it to show ( \frac{1}{2} )</td>
<td></td>
</tr>
<tr>
<td>• Manipulating fraction pieces (bars and/or circles)</td>
<td></td>
</tr>
</tbody>
</table>

### Finding Fractions of a set of objects

- Sharing children out into groups to find \( \frac{1}{2} \) of the group
- Sharing sets of objects into 2 containers and counting contents of one container to find \( \frac{1}{2} \)
- Grouping objects (especially drawn objects as these are harder to share) into 2s and counting the number of groups to find \( \frac{1}{2} \)
- Counting objects to find, for example, the fraction of set of four cubes that are green
- Choosing objects so that a given fraction have a property e.g. \( \frac{1}{2} \) of the counters are red
- Laying out objects equally onto each part of a representation of a fraction e.g. a paper shape or strip and counting the number of objects in the shaded area.

### Representing a Quarter as Proportions of a Shape/Object

- Colouring in 1 part out of 4 in given shapes to find \( \frac{1}{4} \)
- Folding (and colouring) a range of symmetrical paper shapes in half and in half

### Find and recognise ½ of a number of objects

- find half of a set of objects (even number) by sharing them into two containers and counting the contents of one container
- find half of a set of objects (even number) by grouping them into 2s and counting the number of groups
- find half of a set of drawn objects (even number) by sharing them into two groups and counting the number of objects in each group
- find half of a set of drawn objects (even number) by grouping them in 2s and counting the number of groups
- write a statement to represent each calculation, for example, \( \frac{1}{2} \) of 10 = 5
- recognise that when you find half of a set of objects where there are an odd number, you will be left with one object
- know that if objects are in two groups with the same number in each group, then each group represents \( \frac{1}{2} \)

### Find and recognise a quarter of a shape

- find a quarter of a shape with four marked equal parts of same shape
- find a quarter of a shape by splitting it into four equal parts (in different ways if appropriate)
- find a quarter of a shape with eight marked equal parts

### Find and recognise \( \frac{1}{4} \) of these shapes

Show me 1/2 of 12

Always, Sometimes, Never?
You can find one half of 19

Convince me that sharing and grouping give you the same result

Convince me that one half of 20 sweets is bigger than one half of 10 sweets

Show me \( \frac{1}{4} \) of these shapes
<table>
<thead>
<tr>
<th>Finding a Quarter of a set of objects</th>
<th>Representing Fractions as Proportions of a Shape/Object</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Sharing children out into groups to find ¼ of the group</td>
<td>• Colouring in 2 or 3 parts out of 4 in given shapes to find 2/4</td>
</tr>
<tr>
<td>• Sharing sets of objects into 4 containers and counting contents of one container to find ¼</td>
<td>• Folding (and colouring) a range of symmetrical paper shapes in half and in half again to produce quarters, before shading</td>
</tr>
<tr>
<td>• Grouping objects (especially drawn objects as these are harder to share) into 4s and counting the number of groups to find ¼</td>
<td>• Finding two or three quarters of a shape by splitting it into four equal parts (in different ways if appropriate) and shading in two or three parts respectively</td>
</tr>
<tr>
<td>• Counting objects to find, for example, the fraction of set of four cubes that are green</td>
<td>• Find two quarters of a shape with four marked equal parts of same shape</td>
</tr>
<tr>
<td>• Choosing objects so that a given fraction have a property e.g. ¼ of the counters are red</td>
<td>• Find three quarters of a shape with four marked equal parts of the same shape</td>
</tr>
<tr>
<td>• Laying out objects equally onto each part of a representation of a fraction e.g. a paper shape or strip and counting the number of objects in the shaded area.</td>
<td>• Find two or three quarters of a shape by splitting it into four equal parts and counting the number of groups</td>
</tr>
</tbody>
</table>

5. Find and recognise ¼ of a number of objects
- find a quarter of a set of objects (multiple of 4) by sharing them into four containers and counting the contents of one container
- find a quarter of a set of objects (multiple of 4) by grouping them into 4s and counting the number of groups
- find a quarter of a set of drawn objects (multiple of 4) by sharing them into four groups and counting the number of objects in each group
- find a quarter of a set of drawn objects (multiple of 4) by grouping them in 4s and counting the number of groups
- write a statement to represent each calculation, for example, ¼ of 12 = 3
- recognise that sometimes there will be objects leftover when trying to find a quarter
- know that if objects are in four groups with the same number in each group, then each group represents ¼

6. Find and recognise a two or three quarters of a shape
- find two quarters of a shape with four marked equal parts of same shape
- find three quarters of a shape with four marked equal parts of the same shape
- find two or three quarters of a shape by splitting it into four equal parts and shading in two or three parts respectively

Show me ¼ of this square in three different ways
Show me 1/4 of 12
Show me how can you find a 1/4 of these 20 counters
Convince me that half of 10 sweets is the same as 1/4 of 20 sweets
Show me two quarters? three quarters?
Convince me that two quarters is worth the same as one half
What's the same and what's different? half, halve, 1/2, 2/4
<table>
<thead>
<tr>
<th>Finding Fractions of a set of objects</th>
<th>Always, Sometimes, Never?</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Sharing children out into groups to find ( \frac{2}{4}, \frac{3}{4} ) of the group respectively</td>
<td></td>
</tr>
<tr>
<td>• Sharing sets of objects into 4 containers and counting contents of two or three containers to find ( \frac{2}{4}, \frac{3}{4} ) respectively</td>
<td></td>
</tr>
<tr>
<td>• Counting objects to find, for example, the fraction of set of four cubes that are green</td>
<td></td>
</tr>
<tr>
<td>• Choosing objects so that a given fraction have a property e.g. ( \frac{2}{4} ) of the counters are red</td>
<td></td>
</tr>
<tr>
<td>• Laying out objects equally onto each part of a representation of a fraction e.g. a paper shape or strip and counting the number of objects in the shaded area.</td>
<td></td>
</tr>
<tr>
<td>• find two or three quarters of a shape with eight marked equal parts</td>
<td></td>
</tr>
<tr>
<td>• find two or three quarters of a shape with twelve, sixteen or twenty marked equal parts</td>
<td></td>
</tr>
<tr>
<td>• find two or three quarters of a shape with marked equal parts of different shapes</td>
<td></td>
</tr>
<tr>
<td>- begin to realise/notice that two quarters is the same as a half visually</td>
<td></td>
</tr>
<tr>
<td>e.g.</td>
<td></td>
</tr>
<tr>
<td>Always, Sometimes, Never?</td>
<td></td>
</tr>
<tr>
<td>There is no such thing as ( \frac{3}{4} )</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Finding Fractions of a number of objects</th>
</tr>
</thead>
<tbody>
<tr>
<td>7. Find and recognised ( \frac{2}{4}, \frac{3}{4} ) of a number of objects</td>
</tr>
<tr>
<td>• find two or three quarters of a set of objects (multiple of 4) by sharing them into four containers and counting the contents of two or three containers</td>
</tr>
<tr>
<td>• find two or three quarters of a set of drawn objects (multiple of 4) by sharing them into four groups and counting the number of objects in two or three groups</td>
</tr>
<tr>
<td>• find a two or three quarter of a set of objects (multiple of 4) by grouping them in 4s in an array and counting the number of objects in the first two or three columns</td>
</tr>
<tr>
<td>• write a statement to represent each calculation, for example, ( \frac{2}{4} ) of 12 = 6 or ( \frac{3}{4} ) of 8 = 6</td>
</tr>
<tr>
<td>• realise/notice that ( \frac{2}{4} ) of a set of objects give the same result as ( \frac{1}{2} ) of the set of objects</td>
</tr>
<tr>
<td>• know that if objects are in four groups with the same number in each group, then two groups represents ( \frac{2}{4} ) and three groups represents ( \frac{3}{4} )</td>
</tr>
<tr>
<td>8. Find a third of a shape</td>
</tr>
<tr>
<td>• find a third of a shape with three marked equal parts of same shape</td>
</tr>
<tr>
<td>• find a third of a shape by splitting it into three equal</td>
</tr>
<tr>
<td>True or False?</td>
</tr>
<tr>
<td>1/3 is less than 1/4</td>
</tr>
<tr>
<td>Convince me that there is a such a fraction</td>
</tr>
</tbody>
</table>

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<thead>
<tr>
<th>Representing a Third as Proportions of a Shape/Object</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Colouring in 1 part out of 3 in given shapes to find ( \frac{1}{3} )</td>
</tr>
<tr>
<td>3/4 of an amount is larger than 1/2 of an amount</td>
</tr>
<tr>
<td>What's the same and what's different? sharing; grouping</td>
</tr>
<tr>
<td>Always, Sometimes, Never?</td>
</tr>
</tbody>
</table>

| Show me how you can find 2/4 of the 20 counters |
| Show me how you can find 3/4 of the 20 counters |
| What's the same and what's different? sharing; grouping |
| Always, Sometimes, Never? |
| 3/4 of an amount is larger than 1/2 of an amount |
| True or False? |
| 1/3 is less than 1/4 |
| Convince me that there is a such a fraction |

<table>
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<tr>
<th>Representing a Third as Proportions of a Shape/Object</th>
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<tbody>
<tr>
<td>• Colouring in 1 part out of 3 in given shapes to find ( \frac{1}{3} )</td>
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</table>

<p>| Show me how you can find 2/4 of the 20 counters |
| Show me how you can find 3/4 of the 20 counters |
| What's the same and what's different? sharing; grouping |
| Always, Sometimes, Never? |
| 3/4 of an amount is larger than 1/2 of an amount |
| True or False? |
| 1/3 is less than 1/4 |
| Convince me that there is a such a fraction |</p>
<table>
<thead>
<tr>
<th>Finding a Third of a set of objects</th>
<th>Fractions of Lengths</th>
<th>Finding 1/3 of a number of objects</th>
</tr>
</thead>
<tbody>
<tr>
<td>- Sharing children out into groups to find ( \frac{1}{3} ) of the group</td>
<td>- Using a bead string to count out the number and share it into ( \frac{2}{3}/\frac{4}{4} ) equal groups to find the value of ( \frac{1}{2}/\frac{1}{3}/\frac{2}{3}/\frac{3}{4} ) of the number</td>
<td>- Show me 1/3 of 12</td>
</tr>
<tr>
<td>- Sharing sets of objects into ( 3 ) containers and counting contents of one container to find ( \frac{1}{3} )</td>
<td>- Using a paper strip alongside to represent a line and folding it into ( \frac{2}{3}/\frac{4}{4} ) pieces to find the value of ( \frac{1}{2}/\frac{1}{3}/\frac{1}{4} ) of its length</td>
<td>- Show me how you can find 1/3 by sharing</td>
</tr>
<tr>
<td>- Grouping objects (especially drawn objects as these are harder to share) into ( 3 )s and counting the number of groups to find ( \frac{1}{3} )</td>
<td></td>
<td>- Show me how you can find 1/3 by grouping</td>
</tr>
<tr>
<td>- Counting objects to find, for example, the fraction of set of three cubes that are green</td>
<td></td>
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<td>- Choosing objects so that a given fraction have a property e.g. ( \frac{1}{3} ) of the counters are red</td>
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<td>- Laying out objects equally onto each part of a representation of a fraction e.g. a paper shape or strip and counting the number of objects in the shaded area.</td>
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</tbody>
</table>

**9.** Find and recognise 1/3 of a number of objects

- find a third of a set of objects (multiple of 3) by sharing them into three containers and counting the contents of one container
- find a third of a set of objects (multiple of 3) by grouping them into 3s and counting the number of groups
- find a third of a set of drawn objects (multiple of 3) by grouping them in 3s and counting the number of groups
- write a statement to represent each calculation, for example, \( 1/3 \) of 6 = 2
- recognise that sometimes there will be objects leftover when trying to find a third
- know that if objects are in three groups with the same number in each group, then each group represents 1/3

**10.** Find fractions of lengths

- find half/a quarter/a third of a length on a beadstring by splitting the correct number of beads into \( \frac{2}{3}/\frac{4}{4} \) equal groups and counting the size of the first group
- find half of a drawn length (shown alongside a ruler) by splitting the line into two equal parts and counting the contents of one container
- find a third of a shape with six marked equal parts
- find a third of a shape with nine, twelve, ... marked equal parts
- find a third of a shape with marked equal parts of different shapes

**What’s the same and what’s different?**

\( 1/4, 2/4, 3/4, 4/4 \)
**Using Cuisenaire rods to produce lengths that are half/a third/a quarter of another.**
- Using a counting stick to count up and down in halves, thirds and quarters. Beginning to recognise where the wholes appear in this process.

**Using a counting stick to count up and down in halves, thirds and quarters. Beginning to recognise where the wholes appear in this process.**

**Solving Fraction Problems**
- Producing a bar model representation for the fraction in the question and laying objects onto it or colouring it as appropriate

**measuring one part**
- find a quarter of a drawn length (shown alongside a ruler) by splitting the line into four equal parts and measuring one part
- find a third of a drawn length (shown alongside a ruler) by splitting the line into three equal parts and measuring one part
- find two quarters/three quarters of a length on a beadstring by splitting the beads into 4 equal groups and counting the size of the first two/three groups combined
- find two or three quarters of a drawn length (shown alongside a ruler) by splitting the line into four equal parts and measuring the total length of the first two/three parts

**11. Solve problems involving fractions of shapes and sets of discrete objects.**
- read and find a fraction of a number by choosing own concrete or pictorial aids e.g. 1/3 of 15
- create a set of objects with a given fractional property e.g. set of animals where ½ of them have four legs
- find the whole given the fraction e.g. here are 1/3 of my number is 4. What is my number?
- word problems involving fractions e.g. Sam's age is a half of his brother's. His brother is 14. How old is Sam?
- comparison problems e.g. which is greater? ½ of 12 or ¼ of 20?
- comparison word problems e.g. would you rather have ½ of this bar of chocolate or 1/3 of that one?
- find fractions of objects that can be subdivided in different ways e.g. bars of chocolate with 12 squares (3 x 4)
- solving problems with money e.g. find half of 18p (shown as coins)

**Convince me that 1/4 of 12 sweets is less than 1/3 of 12 sweets**

**What's the same and what's different?**
- 1/2, 1/3, 1/4, 2/4

**Always, Sometimes, Never?**
- 2/4 is the same as 1/2
### Further Extension

1. Use the pictures to complete the number sentences.

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<tbody>
<tr>
<td>3/4</td>
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2.  
   a) Jo bought a bag of 12 cherries. Jo ate half the number of cherries in the bag. How many cherries did Jo eat?

   b) Jo bought a bag of cherries. Jo ate half the number of cherries in the bag. Jo had 7 cherries left. How many cherries did Jo buy?

   c) Sam bought a bag of 18 cherries. Sam ate 6 cherries. What fraction of the bag of cherries did Sam eat?

   d) Sam bought a bag of cherries. Sam ate 9 cherries and had 3 left over. What fraction of the bag of cherries did Sam eat?

### Rich and Sophisticated Tasks

Recognise, find, name and write fractions 1/3, ¼, 2/4 and ¾ of a length, shape, set of objects or quantity

NRICH: [Making Longer, Making Shorter](https://nrich.maths.org/2346)

NRICH: [A Bowl of Fruit](https://nrich.maths.org/2333)

### Other Problems

What fraction is the red part of the whole circle?

Explain your reasoning.

### Useful Resources

Fraction Manipulatives - exploring equivalence [http://donnayoung.org/math/fraction.htm](http://donnayoung.org/math/fraction.htm)

Fraction models and support questions - [http://www.annery-kiln.eu/gaps-misconceptions/all-images.html](http://www.annery-kiln.eu/gaps-misconceptions/all-images.html)
3.
\[
\frac{1}{3} \text{ of } 3 = 1 \\
\frac{1}{3} \text{ of } 6 = 2 \\
\frac{1}{3} \text{ of } 9 = 3 \\
\frac{1}{3} \text{ of } 12 = \\
\]
Continue the pattern.
What do you notice?

4.
Which of these diagrams have \( \frac{1}{4} \) of the whole shaded?

![Diagrams]

Explain your reasoning.

5.
Colour in \( \frac{1}{4} \) of each of these grids in a different way. Try to think of an unusual way.

![Grids]

How many squares did you colour each time?

6.
If you count in steps of \( \frac{1}{3} \) starting from 0, how many steps will it take to reach:
2, 4 or 6
What do you notice?
### Misconceptions

Some children do not understand that the parts of a whole must be of equal size. They therefore think that you can find a fraction by dividing the shape up into the correct number of pieces, whether or not these are equally sized. For example, to find a third, they may cut a shape into 3 differently sized pieces.

Note that it is not essential for each part to be the same shape (only the same size) and some children find this difficult to accept. For example, this diagram does show quarters of the whole although the four parts are not all the same shape.

Similarly, children may become used to certain representations and the position of the shaded piece. They do not always appreciate that the position of the shaded piece can vary. For example, these images all show $\frac{1}{4}$

Conceptually, some children do not understand the role of the denominator in telling us the number of parts in the whole (or that the whole is shared into) They therefore often believe that, for example, $\frac{1}{2}$ is smaller than $\frac{1}{4}$ because 2 is less than 4.

Sometimes children read fractions as pieces and do not break down larger pieces into equal parts. They can think of fractions as ratios rather than comparing to the whole. So, for example, $\frac{3}{4}$ can be seen as 1/3.

Children tend to stick to one particular representation of a fraction (often the circular one) and do not recognise other models or images as also being worth the same. Exposure to a wide range of apparatus and pictorial stimuli is required to challenge this.

Sometimes when finding $\frac{2}{4}$ or $\frac{3}{4}$, children incorrectly divide twice. For

### Teacher Guidance and Notes

- This unit precedes the unit on multiplication and division. Therefore, the focus of this unit is on the concepts of fractions as parts of wholes; the process of dividing is less crucial. Note that basic concrete sharing and grouping will have been covered in Stage 1 and so children have some prior knowledge. Teachers should keep the numbers for calculations small to allow children to develop the concepts instead. They should avoid using the division sign. Pupils can then apply these ideas after Unit 9 when they have developed their skills in division further.

- In Stage 2, fractions are referred to as proportions of an amount (i.e. $\frac{1}{2}$ of ...).

- Be aware of the conceptual difference in referring to a fraction of a shape or object using partitioning into equal pieces (for example, half of this shape has been shaded) and referring to finding a fraction of an amount using calculation.

- In Stage 3, children will begin to see fractions as numbers themselves (rather than proportions only) – so leave this development for the next stage.

- Ensure children have lots of opportunity to find fractions of wholes concretely and visually. It is useful to start to prompt children to use an array to ‘organise’ their groups. For example, showing a third of 12 by arranging twelve items into an array with three rows (because we are finding a third) and counting how many columns this creates. This will help make links to division also.

- This is a good time to introduce the bar model as a way of representing unit fractions (and then non-unit examples). You can start this with paper rectangles/strips at this stage and model the bar electronically.

- As with all work on fractions, it is essential that children understand the importance of dividing the quantity or object into equal parts - so emphasise the use of the same size blocks in the bar model.

- Ensure children know that the denominator represents the number of parts in the whole and the numerator represents the number of parts that we are working with e.g. that are shaded/blue/tomatoes etc.

- When exploring representations, do not forget the symbols themselves - it is worth exploring that $\frac{1}{2}$ means 1 divided by 2 and so on. You can see a lovely example of children exploring fractions as a sharing model on YouTube here: [https://www.youtube.com/watch?v=Q-yichde66s&feature=youtu.be](https://www.youtube.com/watch?v=Q-yichde66s&feature=youtu.be)

- You can make connections to children’s skills in doubling when solving reverse problems.
example, to find \( \frac{3}{4} \) they may split into 4 equal pieces to find \( \frac{1}{4} \) and then split into 3 pieces once again, rather than combining three separate quarters together.

There can sometimes be misunderstanding about identical fractions of different ‘wholes’ where the whole is not clear, for example with money e.g. half of 50p.

**Key Assessment Checklist**

1. I can recognise and say the fraction of a shape that has been shaded or the fraction of a set of objects that has a certain feature
2. I can find a half, a third and a quarter of a shape by splitting it into 2, 3 or 4 equal parts
3. I can find two quarters or three quarters of a shape by splitting it into 4 equal parts and shading 2/3 of them.
4. I can represent a third, a quarter and a half using a range of models and images (including the fraction symbols themselves i.e. \( \frac{1}{2}, \frac{1}{3}, \frac{1}{4} \))
5. I can represent two quarters and three quarters using a range of models and images (including the fraction symbols themselves i.e. \( \frac{2}{4}, \frac{3}{4} \))
6. I can find a half, third or a quarter of a number of objects or quantity by sharing into 2, 3, 4 equal groups respectively.
7. I can find a half, third or quarter of a number of objects or quantity by grouping into 2s, 3s or 4s respectively.
8. I can find 2/4 or 3/4 of a number of objects or quantity by first finding 1/4.
9. I can show 1/2, 1/3, 1/4 of a shape or length by dividing it into two, three or four equal parts respectively.
10. I can recognise that the fractions 2/4 and 1/2 are equivalent and show this for a given quantity.
11. I can write simple fractions of quantities to show my calculations e.g. \( \frac{1}{2} \) of 6 = 3 to show finding a half of 6 or \( \frac{1}{3} \) of 15 = 5 to show finding a third of 15.
12. I can solve problems involving fractions of shapes and sets of objects.
## Stage 2

### 12 learning hours

This unit continues pupils' earlier study of arithmetic (and algebra for secondary students).
At Key Stage 1 children are working on multiplication (and division in Stage 2) as a way to represented repeated addition and scaling (and repeated subtraction – grouping - and sharing).
At Key Stage 2 children are developing skills in applying their arithmetic to more complex problems.
At secondary level and in Stage 6, students begin to find unknown values by applying inverse operations. Equations of all types including quadratic and simultaneous are covered in later stages.

### Prior Learning

- solve one-step problems involving multiplication and division, by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher

### Core Learning

- calculate mathematical statements for multiplication and division within the multiplication tables and write them using the multiplication (×), division (÷) and equals (=) signs
- show that multiplication of two numbers can be done in any order (commutative) and division of one number by another cannot
- begin to recall and use multiplication and division facts for the 2, 5 and 10 multiplication tables, including recognising odd and even numbers

**Working towards:**
- recall doubles and halves to 20 (e.g. the pupil know that double 2 is 4, double 5 is 10 and half of 18 is 9)

**Working at expected standard**
- recall and use multiplication and division facts for the 2, 5 and 10 multiplication tables to solve simple problems, demonstrating an understanding of commutativity as necessary (e.g. knowing they can make 7 groups of 5 from 35 blocks and writing 35 ÷ 5 = 7; sharing 40 cherries between 10 people and writing 40 ÷ 10 = 4; stating the total value of six 5p coins) (repeat)

**Working at greater depth:**
- use multiplication facts to make deductions outside known multiplication facts (e.g. a pupil knows that multiples of 5 have one digit of 0 or 5 and uses this to reason that 18 × 5 cannot be 92 as it is not a multiple of 5). (repeat)

### Learning Leads to...

- write and calculate mathematical statements for multiplication and division using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental and progressing to formal written methods
Exemplification

1. a) Write a **multiplication** statement to that this diagram represents:
   ![Multiplication Diagram]

   b) Write a **division** statement that this diagram represents:
   ![Division Diagram]

c) Calculate $5 \times 10$

d) Calculate $20 \div 5$

2. Bob is using the number 4, 2 and 8 to make a fact family.
   Put a cross next to the number sentence that is incorrect.
   
   $2 \times 4 = 8 \quad 4 \times 2 = 8 \quad 8 \div 2 = 4 \quad 4 \div 8 = 2$

3. Here are some number cards:

   ![Number Cards]

   Show how the cards can be arranged to complete a division and a multiplication:

   ![Division and Multiplication Arrangement]
### Practical Multiplication

**•** Arranging objects into equal groups and counting efficiently in 2s, 5s or 10s (to represent repeated addition) – e.g. $3 \times 4$ would be 4 groups of 3 objects (e.g. sweets, animals, books, bean bags, counters, cubes and so on arranged into hoops, lily pads, boxes, bags, and so on) e.g. 2 frogs on each lily pad or 5 shapes in each hoop.

**•** Using Cuisenaire rods (or straws) to scaling an amount/length by making it twice as big/doubling it/making it ten times bigger and so on.

For example, doubling 8

### Abstract Multiplication

**•** Saying the multiplication that has been made. For example, for $2 \times 4$ we would expect to see groups of 2 shown four times and children to practise reading and saying the multiplication as they are building it ‘4 lots of 2’ and ‘4 groups of 2’.

**•** Using an array (with help) to structure the groups of 2/5/10 more formally e.g. $5 \times 3$

**•** Counting efficiently in 2s, 5s or 10s along columns of the array (pointing at the column being counted)

### Fluency

1. Solve practical problems involving multiplication e.g. Roy has 3 buckets with 5 crabs in each. How many crabs does he have altogether?
   - represent the problem concretely or visually using groups or scaling (as implied by the question)
   - find the total by counting (or efficient counting e.g. in 5s)

### Probing Questions

<table>
<thead>
<tr>
<th>Representation</th>
<th>Fluency</th>
<th>Probing Questions</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Practical Multiplication</strong></td>
<td>1. Solve practical problems involving multiplication e.g. Roy has 3 buckets with 5 crabs in each. How many crabs does he have altogether?</td>
<td>What’s the same and what’s different? 5 10ps, 10 5ps, 10 x 5, 5 x 10, 5 lots of 10, 10 lots of 5</td>
</tr>
<tr>
<td><strong>Abstract Multiplication</strong></td>
<td>2. Calculate (abstract) mathematical statements for multiplication e.g. $5 \times 4$</td>
<td>Show me the array for $4 \times 5$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>What’s the same and what’s different? 2 x 6, 12 x 1, 3 x 4, 4 x 2, 6 x 2, 1 x 12</td>
</tr>
</tbody>
</table>
### Simple Word Problems
- Representing a problem using Numicon, groups, beadstrings, arrays, or a number line.
- Matching different representations together to find equivalent pairs.

### Practical Division (Sharing or Grouping depending on the problem’s language)
- Sharing out objects equally into groups. E.g., for $15 \div 3$ there would be 15 items counted and then shared into three piles (divided into 3) and the number in each pile counted.
- Sharing objects more abstractly into groups and linking this to fractions using circles or squares.

### 3. Recognise and solve word problems (out of context) involving multiplication using number sentences e.g.
- **What is twice as many as 8?**
  - choose an appropriate representation
  - solve the problem
  - write a number sentence to represent the problem

### 4. Solve practical problems involving division e.g. Emma has 12 sweets. She shared the sweets between 4 party bags. How many sweets does she put in each bag?
- represent the problem concretely using grouping or sharing (as described by the question)
- find the result by counting the number of groups or number in each group as appropriate

### Convince me that when I multiply an odd number by 2, I get an even answer
- **Always, Sometimes, Never?**
- You can double any number
- **Always, Sometimes, Never?**
- When you multiply an odd number by an odd number you get an odd answer.

### Show me 12 shared between 2 people
- Show me 12 grouped in 2s.
- What’s the same and what’s different?
- Sharing and Grouping
- What’s the same and what’s different?
- 20 shared between 5; 5 shared between 20
<table>
<thead>
<tr>
<th>bar model templates</th>
</tr>
</thead>
</table>
| • Grouping objects into 2s, 3s and so on before counting the number of groups produced  
  e.g. $15 \div 5$ or $15$ grouped into $5$s  |

<table>
<thead>
<tr>
<th>Abstract Division</th>
</tr>
</thead>
</table>
| • Sharing out objects into equally sized groups  
  • Grouping objects and counting the number of groups  
  • Using an array to organise groups  
  e.g. $15 \div 5$ is shown by taking $15$ objects and arranging them in groups (columns) of size $3$ to see how many columns this produces  |

<table>
<thead>
<tr>
<th>Simple Word Problems</th>
</tr>
</thead>
</table>
| • Representing a problem using Numicon, groups, bead strings, arrays, or a number line  
  Examples:  |

| 5. Calculate (abstract) mathematical statements for division  
  e.g. $25 \div 5$  |
|--------------------|
| • represent the statement concretely or visually by sharing  
  • find the result by counting the number in each group  
  • represent the statement concretely or visually by grouping  
  • find the result by counting the number of groups  
  • represent the statement concretely or visually using an array (organised grouping)  
  • find the result by counting the number of columns (groups)  
  • record the result at the end of the number sentence  |

<table>
<thead>
<tr>
<th>6. Recognise and solve word problems (out of context) involving division using number sentences e.g. What is $25$ shared between $5$?</th>
</tr>
</thead>
</table>
| • choose an appropriate representation  
  • solve the problem  
  • write a number sentence to represent the problem  |

| Show me how you can represent $15 \div 5$  
Convince me that $12$ divided by $3$ and $12$ divided into $3$s have the same answer  |

| What's the same and what's different? $10 \div 5$ and $5 \div 10$  
Always, Sometimes, Never?  
When you divide an even number by an even number you get an even answer.  |
### Commutativity
- Using an array to show that, for example, $5 \times 2$ is the same as $2 \times 5$.
- Representing $12 \div 2$ using grouping and then trying to do the same for $2 \div 12$ to show why these calculations do not have the same result.

### Representations $\rightarrow$ Calculations
- Matching pairs of calculations and representations (practically or as images) [Video Example](#) from NCETM.

### Calculations $\rightarrow$ Representations
- Given a representation, suggest a calculation that it represents:
  - groups
  - scaling
  - array

### Convince me
- State/show whether two multiplication or division calculations have the same result.
  - multiplication of two specific numbers
  - division of two specific numbers
  - general principle for multiplication
  - general principle for division
  - use the word ‘commutative’ for multiplication

- Convince me that multiplication is commutative.
  - Always, Sometimes, Never?
  - You get a different answer if you divide in the other order.

- Show me how you can represent $10 \times 4$ in as many ways as possible.
  - Convince me that this array represents $10 \div 2$ and $5 \times 2$.
  - How do these calculations relate?
  - Show me two different calculations that this image could represent:
Fact Families
- Given an array, saying the multiplication it represents
- Given an array, saying the division it represents
- Making an array to show $3 \times 5$ and then representing $15 \div 5$ with an array to discover that you get the same array. Writing the calculations that these represent and then showing $5 \times 3$ as well as $15 \div 3$.

<table>
<thead>
<tr>
<th>Multiplication and Division</th>
<th>Multiplication and Division Facts</th>
</tr>
</thead>
<tbody>
<tr>
<td>- Representing simple problem using trigger words of 'lots of/groups of' or 'times bigger' or 'shared between' or 'shared into groups of' to decide what type of representation</td>
<td>- Using a counting stick to represent counting in 2s, 5s, 10s etc. from 0.</td>
</tr>
<tr>
<td></td>
<td>- Chanting these in order AND pointing to specific multiples in a random order to recall them</td>
</tr>
<tr>
<td></td>
<td>- Removing some labels as time progresses to develop the recall of key facts</td>
</tr>
</tbody>
</table>

11. Begin to recall times table multiplication facts (2s, 5s and 10s)
- By representing the calculation concretely to deduce the answer.
- By representing the calculation visually to deduce the answer.
- By relating the calculation to another known calculation and counting on/back or doubling etc.
- By beginning to recall key facts.

12. Begin to recall times table division facts (2s, 5s and 10s)
- By representing the calculation concretely to deduce the answer.
- By representing the calculation visually to deduce the answer.
- By relating the calculation to another known calculation and counting on/back or doubling etc.
- By beginning to recall key facts.

<table>
<thead>
<tr>
<th>9. Find the fact family for a given multiplication or division</th>
<th>Convince me that if $4 \times 2 = 8$ then $8 \div 2$ must be 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>- given a multiplication, find the answer and then the other three related calculations</td>
<td>Always, Sometimes, Never?</td>
</tr>
<tr>
<td>- given a division, find the answer and then the other three related calculations</td>
<td>If you know that $a \times b = c$ then you can make another three true number sentences about these numbers</td>
</tr>
<tr>
<td>- from a representation e.g. an array</td>
<td>Show me a number that is 5 times bigger than 6</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>10. Ext: Solve a mixture of multiplication and division problems</th>
<th>What’s the same and what’s different?</th>
</tr>
</thead>
<tbody>
<tr>
<td>- know key trigger words for multiplication and division</td>
<td>15, 40, 12, 16, 35, 30, 18, 20</td>
</tr>
<tr>
<td>- recognise whether problem is multiplication or division</td>
<td>30 ÷ 10; 20 ÷ 5; 8 ÷ 4; 15 ÷ 5; 6 ÷ 2; 40 ÷ 10</td>
</tr>
<tr>
<td>- represent the problem concretely or visually</td>
<td>Convince me that $35 \div 5 = 7$ in 3 different ways</td>
</tr>
<tr>
<td>- solve the problem</td>
<td></td>
</tr>
<tr>
<td>- record the problem using a number sentence</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>11. Begin to recall times table multiplication facts (2s, 5s and 10s)</th>
<th>12. Begin to recall times table division facts (2s, 5s and 10s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>- By representing the calculation concretely to deduce the answer.</td>
<td></td>
</tr>
<tr>
<td>- By representing the calculation visually to deduce the answer.</td>
<td></td>
</tr>
<tr>
<td>- By relating the calculation to another known calculation and counting on/back or doubling etc.</td>
<td></td>
</tr>
<tr>
<td>- By beginning to recall key facts.</td>
<td></td>
</tr>
</tbody>
</table>
### Further Extension

1. Which has the most biscuits:
   - 4 packets of biscuits with 5 in each packet, or
   - 3 packets of biscuits with 10 in each packet?

   Explain your reasoning.

2. Find different ways to find the answer to $12 \times 4$.

   Children are expected to use their 2, 5 and 10 times tables to answer this question.

3. Write these addition sentences as multiplication sentences.

   $10 + 10 + 10 + 5 + 5 =$
   $2 + 2 + 2 + 4 =$
   $2 + 2 + 4 + 4 =$
   $5 + 5 + 5 + 2 + 3 =$

4. True or false?

   - $5 \times 4 = 4 \times 5$
   - $5 \times 4 = 10 \times 2$
   - $5 \times 4 = 2 \times 10$

   Explain your reasoning.

   What do you notice?

### Rich and Sophisticated Tasks

Calculate mathematical statements for multiplication and division within the multiplication tables and write them using the multiplication ($\times$), division ($\div$) and equals ($=$) signs

 NRICH: [Ordering Cards] * G
 NRICH: [Which Symbol?] * P
 NRICH: [I’m Eight] * I


Recall and use multiplication and division facts for the 2, 5 and 10 multiplication tables, including recognising odd and even numbers

 NRICH: [Odd Times Even] *** I
 NRICH: [Two Numbers Under the Microscope] ** I
 NRICH: [Even and Odd] * I
 NRICH: [Ring a Ring of Numbers] * G
 NRICH: [More Numbers in the Ring] *** G P
 NRICH: [How Odd] ** I
 NRICH: [Clapping Times] G I
 NRICH: [Double or Halve?] * G
 NRICH: [Always, Sometimes or Never?] * P
Two friends want to buy some marbles and then share them out equally between them. They could buy a bag of 13 marbles, a bag of 14 marbles or a bag of 19 marbles. What size bag should they buy so that they can share them equally? What other numbers of marbles could be shared equally? Explain your reasoning.

<table>
<thead>
<tr>
<th>Misconceptions</th>
<th>Teacher Guidance and Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Children may assume that, since multiplication is commutative, division is commutative and can be done in any order! They may write sentences such as $6 \div 2 = 12$ due to this.</td>
<td>• Children will have encountered multiplication and division verbally and practically in Stage 1 but this is their first encounter in Stage 2, and their first in using the notation themselves.</td>
</tr>
<tr>
<td>Children struggle with the range of language for both multiplication and division - they may incorrectly read $3 \times 4$ as 3 lots of 4 rather than 3 multiplied by 4 (which is actually therefore 4 lots of 3) and so on. This can lead to errors in representing the array correctly (which we want to avoid to enable them to ‘see’ commutativity and to later represent more complex multiplication like $13 \times 4$ in a consistent way).</td>
<td>• They will work on these skills again in Unit 13. Therefore, the focus at this stage is in the processes of building/drawing representations and using these to complete number sentences. It is advised that you use explicit real life examples to introduce each operation but that you focus on getting children strong and quick at representing a given calculation. The opportunity to apply this to more complex contexts and word problems will then come in Unit 13.</td>
</tr>
<tr>
<td>Similarly, they may find it hard to understand what operation they need to use from a word problem because there are so many ways to imply a multiplication or division.</td>
<td>• Initially it is important that children can use and recognise a wide range of representations of multiplication and division calculations.</td>
</tr>
<tr>
<td>Children may not see how an array can be used to support division, only multiplication.</td>
<td>• Arrays are longer term the most useful of the representations as they will form the basis of our later work on multiplication and division. Therefore, it is important that adequate time has been spent getting the array right and recording the number sentences alongside the array. Children should be given opportunities to build their arrays before moving on to drawing these and then eventually the children will be able to just visualise the array and solve the multiplication/division.</td>
</tr>
<tr>
<td>Arrays are also an effective way to explore commutativity – Children to compare the array for $2 \times 6$ with one for $6 \times 2$ to see what they notice. Also explore, what happens when you change the numbers around within a division. Children to recognise that they do not have enough objects to share out or group.</td>
<td>• Arrays are also an effective way to explore commutativity – Children to compare the array for $2 \times 6$ with one for $6 \times 2$ to see what they notice. Also explore, what happens when you change the numbers around within a division. Children to recognise that they do not have enough objects to share out or group.</td>
</tr>
<tr>
<td>Ensure that children can confidently recognise the formal symbols for multiplication and division.</td>
<td>• Ensure that children can confidently recognise the formal symbols for multiplication and division.</td>
</tr>
<tr>
<td>When solving multiplication and division statements try to stick to using examples for from the 2, 5 and 10 times table. However, it is possible for the children to solve multiplications from other times tables if you allow for counting in steps and keep numbers small (e.g. $3 \times 4$)</td>
<td>• When solving multiplication and division statements try to stick to using examples for from the 2, 5 and 10 times table. However, it is possible for the children to solve multiplications from other times tables if you allow for counting in steps and keep numbers small (e.g. $3 \times 4$)</td>
</tr>
</tbody>
</table>
### Key Assessment Checklist

1. I can explain what different multiplication and division number statements mean.
2. I can solve multiplication and division problems using practical equipment to create groups.
3. I can solve multiplication and division problems by scaling and sharing.
4. I can solve multiplication and division problems using arrays.
5. I can use the signs x, ÷, and = to record a multiplication or division number sentence.
6. I can show that if I multiply my numbers in any order I will get the same answer.
7. I can show that if I divide my numbers in a different order I will get a different answer.
8. I can solve multiplication and division problems by recalling known facts.
9. I can decide which calculation to carry out and pursue this to solve a problem in context.
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<tr>
<th>Stage 2</th>
<th>Unit 10: Investigating Statistics</th>
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<tbody>
<tr>
<td>6 learning hours</td>
<td>In this unit children and students explore the collection, representation, analysis and interpretation of data. It covers a range of calculations of central tendency and spread as well as multiple charts and graphs to represent data. As it is the only unit directly exploring statistics, it is critical that children have time to explore the handling data cycle here and to focus sufficient time on interpreting their results.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Prior Learning</th>
<th>Core Learning</th>
<th>Learning Leads to...</th>
</tr>
</thead>
</table>
| ➢ measure and begin to record the following:  
  - lengths and heights  
  - mass/weight  
  - capacity and volume  
  - time (hours, minutes, seconds) | ➢ interpret and construct simple pictograms, tally charts, block diagrams and simple tables  
 ➢ ask and answer simple questions by counting the number of objects in each category and sorting the categories by quantity  
 ➢ ask and answer questions about totalling and comparing categorical data | ➢ interpret and present data using bar charts, pictograms and tables  
 ➢ solve one-step and two-step questions [for example, ‘How many more?’ and ‘How many fewer?’] using information presented in scaled bar charts and pictograms and tables |

<table>
<thead>
<tr>
<th>Exemplification</th>
<th>Vocabulary</th>
</tr>
</thead>
</table>
| 1. a) Some children were asked their favourite flavour ice cream. Make a pictogram for the following information:  
Chocolate: 4  
Strawberry: 3  
Vanilla: 5 | results  
data  
list  
categories  
tally (chart)  
frequency  
table  
pictogram  
key  
symbol | block  
diagram/chart  
construct  
most  
least  
difference  
total  
labels  
axes |
| b) How many children were asked altogether? |  |
| 2. Construct a tally chart to find out how many of each type of badge are in the diagram below: |  |
| a) Which type of badge are there more of?  
b) How many badges are there in total? |  |
| 3. Make a suitable chart or diagram to represent the following data:  
Blue Red Blue Red Yellow Red Red Blue Red Yellow Red Red |  |
<table>
<thead>
<tr>
<th>Representation</th>
<th>Fluency</th>
<th>Probing Questions</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Tally Charts</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>* Completing a class survey. * Making a human tally chart using children as each marker. * Initially do not group the children so they can see how tricky it is to count the totals. * Then group the children in 5s, with every 5th child then acting as a group collector. * Counting aloud in 5s to find the total of each category to show how this is easier.</td>
<td>1. Construct a tally chart using a template * identify different categories * create tallies for quantities less than 5 * create tallies for quantities between 5 and 10 * create tallies for quantities greater than 10</td>
<td>Show me what a tally for 4 looks like. What about 5? 6? 34?</td>
</tr>
<tr>
<td><strong>Frequency Tables</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>* Converting human tally chart into a frequency table by finding the total of each row * Practising organising the data into categories also by recording a new category when a child with a new value appears * When using a written list of data, show children how crossing off each item helps us check we haven't counted any twice.</td>
<td>2. Construct a simple frequency table * Complete a template of a table from a list of data by tallying and finding the total * Complete a frequency table from scratch from a list of data o identify the different categories from a list of data o count number of items in each category o create an ordered table</td>
<td>Show me a list of data with three categories Show me the frequency table for this tally chart</td>
</tr>
<tr>
<td><strong>Reading/Interpreting Frequency Tables</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>* Using a human frequency table to read values. Getting children who fit into the description of the question to stand up and the others to sit down so they can be counted.</td>
<td>3. Reading values from tables * Reading a given value from a table e.g. the number of children who like Red best * Finding the total of all the categories (total frequency) * Finding a subtotal e.g. the number of children who have 1 or 2 brothers or sisters * Finding a total involving ‘more’ or ‘less’ e.g. the number of children who have more than 5 pets * Find the category with the highest (or lowest) frequency * Order the categories from least to greatest frequency</td>
<td>Convince me that there total number of pieces of data is 30 Always, Sometimes, Never? The best way to collect information is to ask your friends</td>
</tr>
<tr>
<td><strong>Pictograms</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>* Using counters on a template to make a pictogram practically first</td>
<td>4. Construct a pictogram * Complete a pictogram on a template with a key where each symbol represents 1 item</td>
<td>Show me how you would represent 3 on this pictogram</td>
</tr>
</tbody>
</table>
| Exploring how 1 counter for 1 item is difficult when there are lots of items and so using 1 counter to represent more items (2 or 5) to develop the idea of keys | Produce own pictogram with a key where each symbol represents 1 item  
- identify the categories  
- choose a suitable symbol to represent items  
- use same symbol in the pictogram  
- space the pictures equally to compare easily  
- include a key  
- Complete a pictogram on a template with a key where each symbol represents 2 items  
- Produce own pictogram with a key where each symbol represents 2 items  
- Complete a pictogram on a template with a key where each symbol represents 5 items  
- Produce own pictogram with a key where each symbol represents 5 items  
- Complete/produce a pictogram where each symbol represents 10 items | True or False?  
You cannot use a symbol to stand for 3 things on a pictogram.  
Always, Sometimes, Never?  
A pictogram is the best way to display data |
| --- | --- | --- |
| Block Charts  
- Complete a block diagram template by colouring the blocks  
- Exploring examples with different sized blocks to discover why it is important that they are all the same size to help compare heights | 5. Construct a block chart  
- colour in blocks on a block diagram template to match a frequency table  
- create own block diagram where axes given but scale must be added | Always, Sometimes, Never?  
A block graph shows what people like or don't like |
| Interpreting pictograms and block charts  
- Children should practice counting each category and writing them down - useful for finding totals and comparing  
- Children should also explore how block charts/pictograms are useful for comparing by counting differences  
- When listing values it is useful to order them for analysis  
- When comparing categories it is useful to use counters to represent values to easily count difference | 6. Organising and comparing  
- Reading a given value from a table e.g. the number of children who like Red best  
- Finding the total of all the categories (total frequency)  
- Finding a subtotal e.g. the number of children who have 1 or 2 brothers or sisters  
- Finding a total involving ‘more’ or ‘less’ e.g. the number of children who have more than 5 pets  
- Find the category with the highest (or lowest) frequency  
- Order the categories from least to greatest frequency | Convinc e me that this pictogram and this block graph come from the same list of data  
What's the same and what's different?  
table, block graph, pictogram, tally, list |
### Further Extension

1. Four children played racing games at break time. Each time they won a game they took a counter.

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
</table>
| Sam   | 🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊📌
| Tom   | 🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊📌
| Sally | 🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊📌
| Ally  | 🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊🍊📌

Present the information in a different way to make it clearer and answer the following questions:
- Who won the most races?
- How many more races did Ally win than Sally?

Does the information answer the question: Who is the fastest runner?

2. What's the same? What's different?

<table>
<thead>
<tr>
<th>Ice creams sold in one week</th>
<th>Cars in the car park on Monday at 10 o'clock</th>
</tr>
</thead>
<tbody>
<tr>
<td>Monday</td>
<td>Red</td>
</tr>
<tr>
<td>Tuesday</td>
<td>Blue</td>
</tr>
<tr>
<td>Wednesday</td>
<td>Black</td>
</tr>
<tr>
<td>Thursday</td>
<td>Silver</td>
</tr>
<tr>
<td>Friday</td>
<td>White</td>
</tr>
<tr>
<td>Saturday</td>
<td>Other</td>
</tr>
</tbody>
</table>

### Rich and Sophisticated Tasks

Interpret and construct simple pictograms, tally charts, block diagrams and simple tables

NRICH: **Sticky Data** * G P
NRICH: **If the World Were a Village** * P I
NRICH: **Plants** ** P
NRICH: **What Shape and Colour?** * G
NRICH: **Carroll Diagrams** * P
NRICH: **Ladybird Count** * P

Ask and answer simple questions by counting the number of objects in each category and sorting the categories by quantity

NRICH: **The Hair Colour Game** ** G P
NRICH: **Mixed-up Socks** ** P I
NRICH: **Sort the Street** * P
NRICH: **Button-up** * P
NRICH: **Beads and Bags** * P

Ask and answer questions about totalling and comparing categorical data

NRICH: **In the Playground** * I
**Misconceptions**

- Children may count a piece of data twice or miss some out when completing a tally chart from a list of data - this can be exacerbated if they have collected their own data and recorded it somewhat haphazardly initially.

- Children sometimes use different symbols for different categories in pictograms, making it hard to compare. They need to use the same symbol consistently throughout the pictogram.

- Children forget to bundle in 5s when completing a tally chart and may not count in 5s to save time when totalling.

- Children use limited vocabulary when describing findings e.g. they say that "category A was the best" when in fact they mean the most popular or the longest or the heaviest etc. They also avoid using the word 'frequency' and instead use total/amount. Children do not always think about the context of the data when they are reading from it. e.g. they assume that the highest number is the best.

**Teacher Guidance and Notes**

- This unit is children's first formal introduction to Statistics.

- The content of the unit focuses on BOTH representing and interpreting data so ensure you divide your time appropriately to give sufficient focus to reading from graphs and using this to answer questions.

- As a minimum, it is recommended that you collect some data from or with the children so that they can relate their tables and charts to this data and make sense of it.

- For pictograms, the pitch here is only for keys where the symbol represents, 1, 2, 5 or 10 items. Note that when it represents 5 or 10 items, significant subdivisions of these are not expected to be represented.

- One possible approach to this unit is to complete the full data handling cycle to collect, record, represent and then analyse the data.

- It is crucial to let children see what happens if you do not organise your data or label your axes and so on so that they realise the importance of doing things properly.

- Try to model correct language e.g. frequency rather than amount or total, represents for the key on pictograms, block diagram/chart rather than bar chart.

**Key Assessment Checklist**

1. I can list the categories of data for a tally chart
2. I can record results in tally chart
3. I can produce a frequency table from a tally chart
4. I can construct a pictogram with one picture per item from a tally chart
5. I can construct a block diagram/chart from a tally chart
6. I can read results from a table, pictogram or block diagram.
7. I can use my readings to answer simple questions about the data
8. I can use my readings to make comparisons with the data
Stage 2  

Unit 11: Visualising Shape

8 learning hours

In this unit children focus on exploring shapes practically and visually. There is an emphasis on sketching, constructing and modelling to gain a deeper understanding of the properties of shapes. It is therefore necessary to secure the practical skills at the same time as using them to explore the shapes in questions. At secondary level students are developing their skills in construction and the language/notation of shape up to the understanding, use and proof of circle theorems.

| Prior Learning | Core Learning | Learning Leads to...
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>➢ recognise and name common 2-D and 3-D shapes, including: - 2-D shapes [for example, rectangles (including squares), circles and triangles] - 3-D shapes [for example, cuboids (including cubes), pyramids and spheres]</td>
<td>➢ identify 2-D shapes on the surface of 3-D shapes, [for example, a circle on a cylinder and a triangle on a pyramid] Working towards • recognise and name triangles, rectangles, squares, circles, cuboids, cubes, pyramids and spheres from a group of shapes or from pictures of the shapes (repeat) Working at expected standard • describe the properties of 2-D and 3-D shapes (e.g. the pupil describes a triangle: it has 3 sides, 3 vertices and 1 line of symmetry; the pupil describes a pyramid: it has 8 edges, 5 faces, 4 of which are triangles and one is a square) (repeat) Working at greater depth • describe similarities and differences of shape properties (e.g. finds 2 different 2-D shapes that only have one line of symmetry; that a cube and a cuboid have the same number of edges, faces and vertices but can describe what is different about them). (repeat)</td>
<td>draw 2-D shapes and make 3-D shapes using modelling materials; recognise 3-D shapes in different orientations and describe them</td>
</tr>
</tbody>
</table>

Exemplification

1. a) Here are four shapes. Each shape has one face shaded. Name the shape of the shaded face.

![Shapes](image1.png)

1. b) Jake is looking at a face of a 3D shape. The face is a triangle. What could Jake’s shape be? Give two possible answers.

<table>
<thead>
<tr>
<th>Vocabulary</th>
</tr>
</thead>
<tbody>
<tr>
<td>flat</td>
</tr>
<tr>
<td>curved</td>
</tr>
<tr>
<td>straight</td>
</tr>
<tr>
<td>round</td>
</tr>
<tr>
<td>vertex/vertices</td>
</tr>
<tr>
<td>corner</td>
</tr>
<tr>
<td>point</td>
</tr>
<tr>
<td>face</td>
</tr>
<tr>
<td>side</td>
</tr>
<tr>
<td>edge</td>
</tr>
<tr>
<td>end</td>
</tr>
<tr>
<td>sort</td>
</tr>
<tr>
<td>surface</td>
</tr>
<tr>
<td>cube</td>
</tr>
<tr>
<td>cuboid</td>
</tr>
<tr>
<td>pyramid</td>
</tr>
<tr>
<td>prism</td>
</tr>
<tr>
<td>sphere</td>
</tr>
<tr>
<td>cone</td>
</tr>
<tr>
<td>cylinder</td>
</tr>
<tr>
<td>circle</td>
</tr>
<tr>
<td>circular</td>
</tr>
<tr>
<td>triangle</td>
</tr>
<tr>
<td>triangular</td>
</tr>
<tr>
<td>square</td>
</tr>
<tr>
<td>rectangle</td>
</tr>
<tr>
<td>rectangular</td>
</tr>
<tr>
<td>pentagon</td>
</tr>
<tr>
<td>hexagon</td>
</tr>
<tr>
<td>octagon</td>
</tr>
<tr>
<td>Representation</td>
</tr>
<tr>
<td>----------------</td>
</tr>
<tr>
<td><strong>Sides and Vertices of 2D Shapes</strong></td>
</tr>
<tr>
<td>• Exploring and handling 2D shapes made of cardboard or plastic to count the sides and vertices</td>
</tr>
<tr>
<td>• Making 2D shapes using geoboards and elastic bands (or dotty paper if visual rather than concrete preferred). For example, making a shape with 3 sides and 3 vertices. Is it possible to make a shape with 3 sides and 4 vertices?</td>
</tr>
<tr>
<td>• Making 2D shapes using geoboards and elastic bands (or dotty paper if visual rather than concrete preferred). For example, making a shape with 3 sides and 3 vertices. Is it possible to make a shape with 3 sides and 4 vertices?</td>
</tr>
<tr>
<td>• Use number of sides/vertices to identify polygons i.e. to say if a shape is a quadrilateral or not or is a pentagon or not</td>
</tr>
<tr>
<td>• Identify whether any of the sides are the same length</td>
</tr>
<tr>
<td><strong>Identifying 2D Shapes</strong></td>
</tr>
<tr>
<td>• Matching the names of shapes to images of the shapes and to solid versions. <a href="#">NRich version of shape cards</a></td>
</tr>
<tr>
<td>• Playing ‘Guess My Shape’ using 20 questions format (can be done nicely using a shape fan so that all children put forward a guess after each new fact is revealed)</td>
</tr>
<tr>
<td>• Playing ‘Guess My Shape’ using 20 questions format (can be done nicely using a shape fan so that all children put forward a guess after each new fact is revealed)</td>
</tr>
<tr>
<td><strong>Naming 3D shapes</strong></td>
</tr>
<tr>
<td>• Going on a shape hunt to find specific shapes in real life e.g. cuboids</td>
</tr>
<tr>
<td>• Making models of shapes using plasticine or using construction materials (e.g. blocks, duplo, multi-link etc)</td>
</tr>
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<td></td>
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<tr>
<td></td>
</tr>
<tr>
<td>Faces, Edges and Vertices of 3D Shapes</td>
</tr>
<tr>
<td>--------------------------------------</td>
</tr>
<tr>
<td>- Exploring and handling 3D shapes using mathematical models as well as everyday objects, especially packaging, to help count the faces, edges and vertices.</td>
</tr>
<tr>
<td>- Printing with 3D shapes to explore the shapes of the faces. Which shapes have square faces? Did you print with any shapes with circular faces? What happens when you print with a sphere? A cylinder?</td>
</tr>
<tr>
<td>- Pulling 3D shapes apart (e.g. packets) to see the 2D shapes that they are made from</td>
</tr>
<tr>
<td>- Counting (and marking off) the vertices (edges/faces) of a 3D shape</td>
</tr>
<tr>
<td>- Making models using straws and balls of modelling clay or equivalent. Then exploring how many straws and connecting balls you need to make a cube? A pyramid?</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>5. Describe the faces, edges and vertices of pyramids and cones</th>
<th>Always, Sometimes, Never? A pyramid has 5 vertices</th>
</tr>
</thead>
<tbody>
<tr>
<td>- sketch or find a pyramid or cone</td>
<td>- True or False? A cone is a pyramid</td>
</tr>
<tr>
<td>- state the number of faces of a pyramid or cone</td>
<td></td>
</tr>
<tr>
<td>- identify whether the faces are flat or curved</td>
<td></td>
</tr>
<tr>
<td>- identify whether any of the faces are the same</td>
<td></td>
</tr>
<tr>
<td>- name the shapes of each face of a pyramid or cone (not curved faces)</td>
<td></td>
</tr>
<tr>
<td>- state/count the number of vertices of a pyramid or cone</td>
<td></td>
</tr>
<tr>
<td>- state/count the number of edges of a pyramid or cone</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>6. Describe the faces, edges and vertices of prisms and cylinders</th>
<th>True or False? A shape can be a prism and a pyramid. Convince me that the number of sides on a cross-section of a prism is 2 less than the number of faces.</th>
</tr>
</thead>
<tbody>
<tr>
<td>- sketch or find a prism or cylinder</td>
<td></td>
</tr>
<tr>
<td>- state the number of faces of a prism or cylinder</td>
<td></td>
</tr>
<tr>
<td>- identify whether the faces are flat or curved</td>
<td></td>
</tr>
<tr>
<td>- identify whether any of the faces are the same</td>
<td></td>
</tr>
<tr>
<td>- name the shapes of each face of a prism or cylinder (not curved faces)</td>
<td></td>
</tr>
<tr>
<td>- state/count the number of vertices of a pyramid or cylinder</td>
<td></td>
</tr>
<tr>
<td>- state/count the number of edges of a pyramid or cylinder</td>
<td></td>
</tr>
<tr>
<td>- given a description of the faces/vertices/edges of</td>
<td></td>
</tr>
<tr>
<td>Further Extension</td>
<td>Rich and Sophisticated Tasks</td>
</tr>
<tr>
<td>-------------------</td>
<td>-----------------------------</td>
</tr>
</tbody>
</table>
| **Identify 2-D shapes on the surface of 3-D shapes** [for example, a circle on a cylinder and a triangle on a pyramid] | **NRICH:** **Cubes** *I*  
**NRICH:** **Shadow Play** ***P*** |

<table>
<thead>
<tr>
<th>Comparing 3D shapes</th>
<th>8. Compare 3D shapes</th>
</tr>
</thead>
</table>
| - Sorting 3D shapes into categories e.g. number of faces using practical table with models of the shapes  
- Arranging 3D shapes into Venn diagrams and Carroll diagrams  
- Playing ‘Guess My Shape’ using 20 Qs format. | - by number of faces  
- by shapes of faces  
- by number of vertices  
- by whether faces are curved or straight  
- by whether they meet a category definition e.g. pyramid or prism |
| **Always, Sometimes, Never?**  
Shapes with square bases are pyramids  
Convince me that there are at least two different shapes with 5 faces.  
Show me how you can order 3D shapes by the number of vertices they have |
2. Jack has made a cube using 12 sticks and 8 balls of modelling clay.

What shape could he make with:
6 sticks and 4 balls of clay?
4 long sticks, 8 short sticks 8 balls of clay?

3. This is a triangle – true or false? Explain your answer.

### Misconceptions
Children may confuse 2D and 3D shapes because of the faces on 3D shapes and the complexity of the concept of a 2D shape.

Children’s understanding of surface can be weak and lead to issues understanding the difference between the shape of a face and the whole shape.

Children may interchange prisms and pyramids because of the vocabulary.

Children do not always realise that cones are not pyramids and cylinders are not prisms.

### Teacher Guidance and Notes
- This unit builds on the learning covered in the Exploring Shape unit from earlier in the year
- In this unit children focus on describing the faces of 3D shapes in detail (but earlier work on recognising and naming 2D and 3D shapes may need to be recapped alongside).
- It is key with this unit to ensure that the classroom environment is rich with examples and that all classroom staff model the technical mathematical language. It is important to encourage children to refine their descriptions so that they too use the mathematical language with ease.

### Key Assessment Checklist
1. I can recognise, name and describe the sides and corners of 2D shapes
2. I can count the faces and vertices on 3D shapes
3. I can recognise and name 3D shapes (including cubes, cuboids, pyramids, prisms, cones, cylinders and spheres)
4. I can describe the faces on a cube and a cuboid.
5. I can describe the faces on a range of pyramids and cones.
6. I can describe the faces on a range of prisms and cylinders
7. I can say how many faces a 3D shape has and which shapes
8. I can describe and sort 3D shapes by their faces
**Stage 2**

Unit 12: Exploring Change

### 7 learning hours

For primary pupils this unit focuses on the measures elements of time and co-ordinates. There is a progression from sequencing and ordering through telling the time formally to solving problems involving time. The co-ordinate work flows in the secondary students’ learning focused on the relationships between co-ordinates. Key objectives include the use of $y=mx+c$ for straight lines, the use of functions and the graphing of more complex functions.

### Prior Learning

- sequence events in chronological order using language [for example, before and after, next, first, today, yesterday, tomorrow, morning, afternoon and evening]
- recognise and use language relating to dates, including days of the week, weeks, months and years
- tell the time to the hour and half past the hour and draw the hands on a clock face to show these times

### Core Learning

- compare and sequence intervals of time
- tell and write the time to five minutes, including quarter past/to the hour and draw the hands on a clock face to show these times
- know the number of minutes in an hour and the number of hours in a day.

**Working at expected standard**

- read the time on the clock to the nearest 15 minutes

**Working at greater depth:**

- read the time on the clock to the nearest 5 minutes

### Learning Leads to...

- tell and write the time from an analogue clock, including using Roman numerals from I to XII, and 12-hour and 24-hour clocks
- know the number of seconds in a minute and the number of days in each month, year and leap year
- compare durations of events [for example to calculate the time taken by particular events or tasks]
- estimate and read time with increasing accuracy to the nearest minute; record and compare time in terms of seconds, minutes and hours; use vocabulary such as o’clock, a.m./p.m., morning, afternoon, noon and midnight

### Exemplification

1. Write these times in order from shortest to longest

| 20 minutes | 3 hours | 100 minutes | 1 hour | 15 minutes |

2. a) Write the time shown on these clocks:

![Clocks](image-url)
b) Draw hands on these clock faces to show these times:

<table>
<thead>
<tr>
<th>Ten past five</th>
<th>Quarter to eleven</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

3. a) How many hours are there in one day?
b) How many minutes are there in one hour?

<table>
<thead>
<tr>
<th>Units of Time</th>
<th>Representation</th>
<th>Fluency</th>
<th>Probing Questions</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Counting along with a clock as it ticks a minute to see how many seconds there are in 1 minute</td>
<td>1. Know the relationship between (seconds,) minutes, hours and days</td>
<td>Show me a time that is shorter than 10 minutes</td>
</tr>
<tr>
<td></td>
<td>Counting in 5s around a clock face to discover that there are 60 seconds in a minute and 60 minutes in an hour.</td>
<td>• know the number of minutes in an hour</td>
<td>Show me a time that is longer than 1 hour</td>
</tr>
<tr>
<td></td>
<td>Estimating a minute by putting heads down on the desk and sitting up silently when they think 1 minute has gone.</td>
<td>• know the number of hours in a day</td>
<td>Convince me that 100 minutes is not an hour</td>
</tr>
<tr>
<td>Telling the Time</td>
<td>Labelling a clock with key words and multiples of 5 up to 55 to link to counting in 5s. Then counting round the clock and moving the hands to match e.g. 1:00, 1:05, 1:10, 1:15, .....</td>
<td>2. Tell the time to five minutes using 12-hour format (from an analogue clock)</td>
<td>Show me two times which are an hour apart</td>
</tr>
<tr>
<td></td>
<td>• recap: count in 5s to 60</td>
<td>• recall: count in 5s to 60</td>
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</tr>
<tr>
<td></td>
<td>• read a time from a clock in 12 hour format e.g. 7:25</td>
<td>• move the hands on a clock face to represent a 12-hour time</td>
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<tr>
<td></td>
<td>• move the hands on a clock face to represent a 12-hour time</td>
<td>• draw hands on a clock face to represent a 12-hour time</td>
<td></td>
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<tr>
<td></td>
<td>• draw hands on a clock face to represent a 12-hour time</td>
<td>3. Tell the time to quarter past (analogue clock)</td>
<td>Show me a time which is later in the day</td>
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</tbody>
</table>
### Time Sequencing

<table>
<thead>
<tr>
<th>Time Sequencing</th>
<th>Table</th>
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</thead>
</table>
| Representing a time (including those in excess of 60 minutes) by counting in 5s and using a clock rotation to convert to hours and minutes. | **4.** Tell the time to quarter to hour (analogue clock)  
- tell the time to quarter to the hour  
- associate quarter to with ... : 45  
- move/draw hands on a clock face to show a time of quarter to an hour | Convince me that quarter to ten is before quarter past ten which is before half past ten  
What's the same and what's different? quarter to four; 15 minutes past four; quarter past four |
| Using a number line to represent and position times to support ordering | **5.** Tell the time to the nearest five minutes (analogue clock)  
- tell the time as a multiple of 5 past the hour (five past, ten past, twenty past and twenty-five past)  
- tell the time as a multiple of 5 to the hour (five to, ten to, twenty to, twenty-five to)  
- tell the time from a mixed selection of past and to times (including quarters and halves) | Convince me that quarter past seven is the same as 15 minutes past seven  
What's the same and what's different? 5:30; twenty past five; quarter past five; 5:20, half past five, 5:15 |
| **6.** Show the time to the nearest five minutes (analogue clock)  
- move the hands on a clock to represent a past time (five, ten, quarter, twenty, twenty-five, half)  
- move the hands on a clock to represent a to time (five, ten, quarter, twenty, twenty-five)  
- draw the hands on a clock to represent a given time | Show me two times which are an hour apart on your clock.  
Show me how you would show five past six on this clock | Show me how you would order these time intervals by size  
15 minutes, 1 hour, 1 day, 2 minutes, 100 minutes  
Convince me that these intervals are in size order: 5 minutes, half an hour, 1 hour, 90 minutes |
| **7.** Compare and order lengths of time  
- compare two times in minutes  
- order three or more times in minutes  
- compare two times in hours and minutes  
- order three or more times in hours and minutes  
- find the interval between two times | **8.** Compare and order lengths of time  
Convince me that these intervals are in size order: 5 minutes, half an hour, 1 hour, 90 minutes |
Making timelines to show events in sequence and with more accurate time positioning

Further Extension

1. Which of these clock faces shows a time between 5 o'clock and 7 o'clock?

2. Jack says, 'There isn't any point in having a minute hand on a clock because I can still tell the time without it.'

   Do you agree with him?

   Explain your answer.

Rich and Sophisticated Tasks

Tell and write the time to five minutes, including quarter past/to the hour and draw the hands on a clock face to show these times

- NRICH: What's the Time? * P
- NRICH: Stop the Clock *** G

Know the number of minutes in an hour and the number of hours in a day

- NRICH: Matching Time * G

Misconceptions

Children will encounter a huge range of vocabulary in this topic and may struggle to link the words to the scale of time they represent— for example, when trying to order times they need to have a sense of minutes, hours, days, weeks, months

Teacher Guidance and Notes

- This unit builds on the work in Stage 1 on telling the time to the hour and half hour.
- The expectation is that children can tell the time to 5 minutes (and that...
etc. but this may not come naturally.

Children may still confuse the hands of the clock and the two scales for hours and minutes. This can cause them to invert a time. For example, they may read the correct time of ‘ten past eight’ as ‘eight past two’ or even ‘forty past two’.

Children may also struggle when the hour hand moves between whole hours to see which hour it is nearest to.

The concept of past and to the hour can confuse children in terms of deciding which hour to reference. Children particularly may incorrectly tell ‘to’ times using the previous hour. For example, they may read the time of ‘ten to eleven’ incorrectly as ‘ten to ten’.

Some children may not realise that when the minutes are past the hour, the minute hand must be carefully positioned in relation to how many minutes past the hour it is and not point to the hour.

Some pupils may presume that the decimalised number system applies to time and hence assume there are 100 seconds in a minute or 100 minutes in an hour and so on.

those working at greater depth can approximate times between these intervals to the nearest 5 minutes).

- Some research indicates that children find it easier to tell the time using 12-hour format first (rather than the fractional language and past/to complexities that we generally commence with). This is particularly true for children with certain specific learning needs. Therefore, it is suggested in the fluency steps above that children commence by telling the time using 12-hour format as an hour and number of minutes up to 55 (despite the direct reference to 12-hour time not appearing in the national curriculum until Stage 3).

- More information about this approach can be found in this article on Charlotte’s clock

- As in Stage 1, try to incorporate work on time into daily routines to build children’s knowledge, recall and confidence.

- It can be useful to have 2 scales on the clock in different colours: one representing hours and one representing minutes.

- Include references to the durations of activities e.g. 5 minutes or an hour to help build children’s sense of time (although the calculation of durations is not required until Stage 3)

Key Assessment Checklist

1. I can compare and order intervals of time.
2. I can tell the time to quarter past the hour.
3. I can tell the time to quarter to the hour.
4. I can show the time to the quarter hour on a clock face.
5. I can tell the time to 5 minute intervals.
6. I can show times on a clock face to 5 minutes.
7. I can say the number of minutes and hours in a day.
8. I can use a timeline to order events.
### Stage 2

#### Unit 13: Proportional Reasoning

**7 learning hours**

In this unit pupils explore proportional relationships, from the operations of multiplication and division on to the concepts of ratio, similarity, direct and inverse proportion.

For primary pupils in Stages 1-3, this is focused on developing skills of division. Stages 4 and 5 revisit the whole of calculation to broaden to all four operations in a range of contexts and combination problems; the emphasis here is really on representing and then solving a problem using their calculation skills, not just calculating alone. In Stage 6 the real underpinning concepts of proportion and ratio develop.

Secondary pupils begin to formalise their thinking about proportion by finding and applying scale factors, dividing quantities in a given ratio and fully investigating quantities in direct or inverse proportion, including graphically.

#### Prior Learning

- solve one-step problems involving multiplication and division, by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher

#### Core Learning

- recall and use multiplication and division facts for the 2, 5 and 10 multiplication tables
- solve problems involving multiplication and division, using materials, arrays, repeated addition, mental methods, and multiplication and division facts, including problems in contexts

**Working towards:**

- recall doubles and halves to 20 (repeat)

**Working at expected standard**

- recall and use multiplication and division facts for the 2, 5 and 10 multiplication tables to solve simple problems, demonstrating an understanding of commutativity as necessary (e.g. knowing they can make 7 groups of 5 from 35 blocks and writing 35 ÷ 5 = 7; sharing 40 cherries between 10 people and writing 40 ÷ 10 = 4; stating the total value of six 5p coins)

**Working at greater depth:**

- determine remainders given known facts (e.g. given 15 ÷ 5 = 3 and has a remainder of 0, pupil recognises that 16 ÷ 5 will have a remainder of 1; knowing that 2 × 7 = 14 and 2 × 8 = 16, pupil explains that making pairs of socks from 15 identical socks will give 7 pairs and one sock will be left).

- solve word problems that involve more than one step (e.g. which has the most biscuits, 4 packets of biscuits with 5 in each packet or 3 packets of biscuits with 5 in each packet?).

#### Learning Leads to...

- recall and use multiplication and division facts for the 3, 4 and 8 multiplication tables
- write and calculate mathematical statements for multiplication and division using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental and progressing to formal written methods
- solve problems, including missing number problems, involving multiplication and division, including positive integer scaling problems and correspondence problems

### Exemplification

1. Complete the missing numbers:
   - a) \( 7 \times 2 = \ldots \)
   - b) \( 5 \times \ldots = 20 \)
   - c) \( 60 \div 10 = \ldots \)
   - d) \( \ldots \div 2 = 9 \)

### Vocabulary

- multiply
- \( x \)
- divide
- column
2.  
   a) A teacher puts the children in a class into groups of 5. Altogether there are 30 children in the class. How many groups will the teacher make?  
   b) Matt is buying balloons for a party. The balloons come in packs of 10. Matt buys 7 packets. How many balloons will he have for the party?  
   c) Jessie is calculating $2 + 2 + 2 + 2 + 2 + 2 + 2 + 2 + 2 + 2$. How can Jessie work this out more quickly?  

<table>
<thead>
<tr>
<th>Representation</th>
<th>Fluency</th>
<th>Probing Questions</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Multiplication and Division by 2</strong></td>
<td>1. Recall and use multiplication and division facts for the two times table</td>
<td>What’s the same and what’s different?</td>
</tr>
<tr>
<td></td>
<td>• Arranging objects into equal groups and counting efficiently in 2s</td>
<td>2 x 6, 6 x 2; 24 ÷ 2</td>
</tr>
<tr>
<td></td>
<td>(e.g. sweets, animals, books, bean bags, counters, cubes and so on arranged into hoops, lily pads, boxes, bags, and so on)</td>
<td>Convince me that if I know $13 \times 2 = 26$, I also know three other facts.</td>
</tr>
<tr>
<td></td>
<td>e.g. 2 frogs on each lily pad or 5 shapes in each hoop</td>
<td></td>
</tr>
<tr>
<td></td>
<td>• Sharing a set of objects into 2 groups and counting the number in each group</td>
<td></td>
</tr>
<tr>
<td></td>
<td>• Grouping a set of objects into 2s and counting the number of groups</td>
<td></td>
</tr>
<tr>
<td></td>
<td>• Using an array to represent a multiplication e.g. $2 \times 3$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>• Using an array to represent a division e.g. $10 \div 2$</td>
<td></td>
</tr>
<tr>
<td><strong>Multiplication and Division by 10</strong></td>
<td>2. Recall and use multiplication and division facts for the ten times table</td>
<td>True or False? A number that divides by 10 will divide by 2</td>
</tr>
<tr>
<td></td>
<td>• Arranging objects into equal groups and counting efficiently in 10s</td>
<td>Convince me that the missing number</td>
</tr>
<tr>
<td></td>
<td>(e.g. sweets, animals, books, bean bags, counters, cubes and so on arranged into hoops, lily pads, boxes, bags, and so on)</td>
<td></td>
</tr>
</tbody>
</table>
### Multiplication and Division by 10
- Sharing a set of objects into 10 groups and counting the number in each group
- Grouping a set of objects into 10s and counting the number of groups
- Using an array to structure the groups of 10 more formally e.g. 10 × 3
- Using an array to represent a division e.g. 40 ÷ 10

### Multiplication and Division by 5
- Arranging objects into equal groups and counting efficiently in 5s (e.g. sweets, animals, books, bean bags, counters, cubes and so on arranged into hoops, lily pads, boxes, bags, and so on)
- Sharing a set of objects into 5 groups and counting the number in each group
- Grouping a set of objects into 5s and counting the number of groups
- Using an array to structure the groups of 5 more formally e.g. 5 × 3
- Using an array to represent a division e.g. 20 ÷ 5

### Multiplication and Division Statements
- Using number cards to produce a number sentence to represent a concrete or visual image of multiplication and division
- Using number cards and rearranging these to form

### Recall and use multiplication and division facts for the five times table
- Count in 5s from 0 (forwards and backwards)
- Represent a multiplication by 5 concretely or visually
- Represent a division by 5 concretely or visually
- Complete a missing answer for a multiplication e.g. 7 × 5 = 35
- Complete a missing answer for a division e.g. 60 ÷ 5 = 12
- Find a missing number in a multiplication statement e.g. 5 × 5 = 25
- Find a missing number in a division statement e.g. 5 × 5 = 25
- Give the fact family for a multiplication or division by 5

### What’s the same and what’s different?
- Five 2ps, Ten 1ps, 2 × 5, 5 × 2, 1 lot of 10, 2 lots of 5; Two 5ps

### What’s the same and what’s different?
- A number that is a multiple of 5 and a number that is a multiple of 10

### Convince me that the missing number is 6
- ∎ × 5 = 30

### Create multiplication and division statements
- Produce a concrete/visual representation for a given multiplication or division statement
- Produce an array for a given multiplication or division statement

### Show me how you would write a multiplication sentence for ‘the number that is twice as big as 8’
- ∎ × 5 = 10

### Show me a number sentence you can
## True Statements

- Write a multiplication statement for a given concrete/visual representation (e.g. an array)
- Write a division statement for a given concrete/visual representation (e.g. an array)
- Given number cards and symbols, create correct multiplication and division statements
- Say if a given multiplication or division statement is true or false and justify this

### Make Using 5, 3 and 15

Show me four number sentences that can be made using 4, 5 and 20 and the symbols for multiplication, divide and equals.

## Solving Problems

### Using the Bar Model to Represent Multiplication and Division Problems to Help Decide What Calculation to Complete

- For example: Matt is buying balloons for a party. The balloons come in packs of 10. Matt buys 7 packets. How many balloons will he have for the party?

![Diagram](image1.png)

- For example: A teacher puts the children in a class into groups of 5. Altogether there are 30 children in the class. How many groups will the teacher make?

![Diagram](image2.png)

### Solve Worded Multiplication Problems

- Draw a bar model to represent the word problem
- Represent a word problem concretely or visually (e.g. with groups or scaling)
- Make or draw an array to represent a multiplication problem
- Write a mathematical statement to show the calculation needed
- Solve the problem and complete the mathematical sentence to show the answer

Show me a bar model that represents ‘5 groups of 8’

Show me a bar model that represents the length that is 10 times as long as 3cm.

### Solve Worded Division Problems

- Draw a bar model to represent the word problem
- Represent a word problem concretely or visually (e.g. by sharing/grouping)
- Make or draw an array to represent a division problem
- Write a mathematical statement to show the calculation needed
- Solve the problem and complete the mathematical sentence to show the answer

Show me a bar model that represents ‘35 shared between 5 people’

Show me a bar model that represents ‘16 shared into 2s’

### Identify the Operation Required to Solve a Multiplication or Division Problem

- Know key trigger words for multiplication and division
- Recognise whether problem is multiplication or division
- Represent the problem concretely or visually

What’s the same and what’s different?

**Shared Between 5**
- 5 Lots of
- 5 Times as Big as
  - Divided into 5s
  - Multiplied by 5
<table>
<thead>
<tr>
<th>Further Extension</th>
<th>Rich and Sophisticated Tasks</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Sally buys 3 cinema tickets costing £5 each. How much does she spend? Write the multiplication number sentence and calculate the cost. If Sally paid with a £20 note, how much change would she get?</td>
<td>Recall and use multiplication and division facts for the 2, 5 and 10 multiplication tables, including recognising odd and even numbers</td>
</tr>
<tr>
<td>2. Together Rosie and Jim have £12. Rosie has twice as much as Jim. How much does Jim have? The bar model can be helpful in solving these types of problems. Rosie [ ] = £12 Jim [ ] 12 ÷ 3 = 4 Jim has £4</td>
<td>NRICH: Odd Times Even *** I NRICH: Two Numbers Under the Microscope ** I NRICH: Even and Odd * I NRICH: Ring a Ring of Numbers * G NRICH: More Numbers in the Ring *** G P NRICH: How Odd ** I NRICH: Clapping Times * G I NRICH: Double or Halve? * G NRICH: Always, Sometimes or Never? * P</td>
</tr>
<tr>
<td>3. Two friends share 12 sweets equally between them. How many do they each get? Write this as a division number sentence. Make up two more sharing stories like this one.</td>
<td>Solve problems involving multiplication and division, using materials, arrays, repeated addition, mental methods, and multiplication and division facts, including problems in contexts</td>
</tr>
</tbody>
</table>
### Misconceptions

Children sometimes struggle with the range of language for both multiplication and division - they may incorrectly read $3 \times 4$ as 3 lots of 4 rather than 3 multiplied by 4 (which is actually therefore 4 lots of 3) and so on. This can lead to errors in representing the array correctly (which we want to avoid to enable them to 'see' commutativity and to later represent more complex multiplication like $13 \times 4$ in a consistent way).

Children may not see how an array can be used to support division, only multiplication. They may invert their array accidentally.

Children tend to use the 'lots of' representation [repeated addition] of multiplication much more than scaling. Similarly, they may use sharing more than grouping (which makes using an array for division harder).

When dividing children sometimes muddle the divisor and the dividend and so try to divide 'the wrong way round'.

Children tend to stick to their favourite representations - they may overly rely on, for example, numicon when a bead string or an array could be more helpful.

Children can mistake and record the adding and multiplying symbol.

Children may not recognise key trigger words for multiplication and division and so use the wrong operation when solving a word problem.

Children may make errors if multiplication facts are not secure.

### Teacher Guidance and Notes

- This unit builds on the earlier work on multiplication and division completed in Unit 9. This previous work focused on the conceptual understanding of the processes of multiplication and division as well as the recording of number sentences.

- The focus here is on the recall of the key multiplication tables and the use of these to solve calculations immediately (without working out) as well as on the identification and solution of word and other problems. It is expected that the multiplication and division facts for 2s, 5s and 10s will be a focus of mental maths throughout the academic year and so, by this stage, are more secure.

- If children are secure on the earlier work then you can move on to a problem solving focus quickly. However, if you need to revisit the processes and meanings of multiplication and division, there is the opportunity to do so.

- It is important to use both the representations of arrays and the bar model within this unit in order to lay the foundations for further use in Stage 3 and beyond.

- Note that in Stage 2, we need to formalise what has been learnt in Stage 1 and hence there is greater focus on using correct mathematical language and symbols.

- You should in general use examples of problems based within the 2, 5 and 10 multiplication tables, although there may be the opportunity to go beyond this for children working at greater depth.

- Commutativity is covered in Unit 9, but is used and presumed here by expecting children to be able to recognise the links between related calculations.

### Key Assessment Checklist

1. I can recall and complete multiplication and division facts for the 2 times table.
2. I can recall and complete multiplication and division facts for the 10 times table.
<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
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</thead>
<tbody>
<tr>
<td>3.</td>
<td>I can recall and complete multiplication and division facts for the 5 times table.</td>
</tr>
<tr>
<td>4.</td>
<td>I can represent and solve multiplication word problems</td>
</tr>
<tr>
<td>5.</td>
<td>I can represent and solve division word problems</td>
</tr>
<tr>
<td>6.</td>
<td>I can create multiplication and division statements to represent problems and concrete/visual representations</td>
</tr>
<tr>
<td>7.</td>
<td>I can represent a problem with practical equipment or a picture to help me decide which calculation to carry out.</td>
</tr>
<tr>
<td>8.</td>
<td>I can use this to solve a problem in context.</td>
</tr>
</tbody>
</table>
## Stage 2

### Unit 14: Describing Position

In this unit pupils explore how we can communicate position and movement mathematically. They look at transformations from simple turns to reflection/rotation/enlargement/translations up to similar shapes generated by enlargements, co-ordinate systems and ultimately vectors.

<table>
<thead>
<tr>
<th>Prior Learning</th>
<th>Core Learning</th>
<th>Learning Leads to...</th>
</tr>
</thead>
<tbody>
<tr>
<td>✓ describe position, direction and movement, including whole, half, quarter and three-quarter turns</td>
<td>✓ use mathematical vocabulary to describe position, direction and movement, including movement in a straight line and distinguishing between rotation as a turn and in terms of right angles for quarter, half and three-quarter turns (clockwise and anti-clockwise)</td>
<td></td>
</tr>
</tbody>
</table>

### Exemplification

<table>
<thead>
<tr>
<th>Vocabulary</th>
<th>Where</th>
<th>Position</th>
<th>Above</th>
<th>Below</th>
<th>Next to</th>
<th>Beside</th>
<th>Under</th>
<th>Inside</th>
<th>Outside</th>
<th>In front of</th>
<th>On top of</th>
<th>To the left of</th>
<th>To the right of</th>
<th>Behind</th>
<th>Between</th>
<th>Left</th>
<th>Right</th>
<th>Forward</th>
<th>Backwards</th>
<th>Turn</th>
<th>Half</th>
<th>Quarter</th>
<th>Three quarter</th>
<th>Clockwise</th>
<th>Whole</th>
<th>Anticlockwise</th>
<th>(north)</th>
<th>(east)</th>
<th>(south)</th>
<th>(west)</th>
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1. Look at this picture and complete the sentences:

   **a)**
   - (i) The rectangle is ........ the square
   - (ii) The arrow is ............. the hexagon and ............... the circle.
   - (iii) The ............... is between the rectangle and the cuboid.

   ![Image of a rectangle, circle, arrow, hexagon, and cuboid]

   ![Image of a rectangle and a cuboid]

2. This picture of a dog is rotated three quarters of a turn clockwise.

   ![Image of a rotated dog]

   Which of these images show what the picture will look like afterwards?

   ![Images of rotated dogs]
c) Give instructions to get from the chicken to the sheep (you must stick to the paths)

![Diagram of a grid with a chicken and a sheep]  

<table>
<thead>
<tr>
<th>Representation</th>
<th>Fluency</th>
<th>Probing Questions</th>
</tr>
</thead>
</table>
| **Position**   | 1. Use mathematical language to describe the position of an object  
|                | • find an object given a description of its position  
|                |   • above, below, under, on top of  
|                |   • in front of, behind  
|                |   • next to, beside  
|                |   • to the left of, to the right of  
|                |   • between  
|                |   • inside, outside  
|                | • place an object given a description of its position  
|                | • give a description of a the position of an object  
|                | What's the same and what's different? above; left; right; below; next to; under  
|                | Show me a diagram where:  
|                | The square lies above the triangle.  
|                | The circle is next to the triangle.  
|                | The square is between the hexagon and the triangle.  
|                | Always, Sometimes, Never? Something that is to my left is also to your left  
| **Movement**   | 2. Use mathematical language to describe a movement in a straight line  
|                | • carry out a movement given instructions  
|                |   • forwards, backwards  
|                |   • to the left, to the right  
|                | • describe a movement  
|                | • producing a set of instructions combining movements in straight lines on a grid e.g. two squares forward and 1 square left  
|                | Show me one way to get from A to B on the diagram ... a different way  
|                | What's the same and what's different? up; down; forwards; backwards; left; right |
### Turn
- Turning (people/themselves) through quarter turns and chanting ¼ turn, ½ turn, ¾ turn, whole turn and so on. Changing direction from clockwise to anticlockwise. Using one arm as a point to show the start and end point of each turn. This can also be done with clock hands.
- Using geostrips to produce different angles and turns
- Rotating images to explore what they look like in different orientations
- Exploring a compass to see the links between compass points and turns.

### 3. Use mathematical language to describe a turn (direction and size)
- recognise and carry out a whole turn
- recognise and carry out a half turn (in either direction)
- recognise and carry out a quarter turn (in either direction)
- recognise and carry out a three quarter turn
- use the words clockwise and anticlockwise to describe and carry out turns
- understand a diagram showing a turn with an arrow
- draw a diagram to show a given turn with an arrow
- recognise that the direction does not matter for a half turn
- understand that a ¼ turn in one direction has the same effect as a ¾ turn in the other direction
- understand that a ¼ turn clockwise is the same as turning right and ¼ turn anticlockwise is the same as turning left
- understand that other turns are possible between the quarter turns

### Show me
- ... using your body
- ... using your geostrips:
- ... using the clock hands
- ... using your mini-whiteboard (ie draw)
  - a quarter turn
  - a quarter turn clockwise
  - a quarter turn anticlockwise
  - a half turn
  - a half turn clockwise
  - a whole turn
  - a three quarter turn anticlockwise

Convince me that a half turn clockwise and half turn anti-clockwise will be in the same position

Convince me that a quarter-turn clockwise is the same as a three-quarter turn anti-clockwise

What’s the same and what’s different?
quarter-turn clockwise; quarter turn anticlockwise; three quarter turn clockwise; three quarter turn anticlockwise
### Directions

- Following instructions to find the treasure e.g. walk forwards 10 steps. Then turn a quarter turn to the right.
- Giving their own instructions to find an item to a partner or teacher.
- Using a simple map to follow and create instructions for journeys e.g. give instructions to get from the cat to the chicken.
- Using Beebots or Scratch to experiment with making other objects follow paths and carry out turns.

### 4. Interpret directions for a simple journey

- follow simple directions involving forwards, backwards and turns left and right in a practical situation.
- follow simple directions involving turns (¼, ½, ¾) in a practical situation.
- follow more complex sets of directions (3 or more steps) in a practical situation.
- follow directions on a map (i.e. visually rather than practically).

### 5. Create directions for a simple journey

- produce simple directions involving forwards, backwards and turns left and right in a practical situation.
- produce simple directions involving turns (¼, ½, ¾) in a practical situation.
- produce more complex sets of directions (3 or more steps) in a practical situation.
- produce directions using only a map (i.e. visually rather than practically).

### Further Extension

1. Use mathematical vocabulary to describe position, direction and movement, including movement in a straight line and distinguishing between rotation as a turn and in terms of right angles for quarter, half and three-quarter turns (clockwise and anti-clockwise).

NRICH:
- Turning Man * I
- Walking Round a Triangle * P
- Triangle Animals ** P
- Cover the Camel * P

### Rich and Sophisticated Tasks

Always, Sometimes, Never?
There are two ways to do any turn

What's the same and what's different?
• to the right; to the left; clockwise; anticlockwise

Convince me that going forwards 4 squares, then turning 90 degrees clockwise and then going forwards 3 squares has the same result as turning 90 degrees clockwise, then going forwards 3 squares, then turning 90 degrees anticlockwise and going forward 4 squares.

Show me how you would go round this track using mathematical language.
2. (NRich Task: Coloured Squares)

Use the grid to help you complete this table.

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>trees</strong></td>
<td>B2</td>
</tr>
<tr>
<td><strong>slide</strong></td>
<td></td>
</tr>
<tr>
<td><strong>seesaw</strong></td>
<td>A3</td>
</tr>
</tbody>
</table>

2. (NRich Task: Coloured Squares)

Use these clues to colour each shape:

- Blue is between green and red
- Orange is below green
- Yellow is to the left of both purple and orange

<p>| | |</p>
<table>
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</table>

Misconceptions

Children may still be confusing left and right directions, particularly when starting from a position with a non-standard orientation.

Children may confuse clockwise with anti-clockwise directions or possibly fail to understand that there are two possible directions of turn.

Some children always turn in a clockwise direction, regardless of the instructions.

Some children do not recognise that all the points of a shape move in the same direction and the same distance during a linear movement (translation) and may distort their shape. Similarly they may do this under a rotation.

Teacher Guidance and Notes

- This unit builds on the work of Stage 1 and formalises the use of the language.
- You can relate clockwise and anticlockwise movement to a clock and use this a mainstay of teaching in the unit - children should be able to describe the relationships between the hands and the movement of them in terms of rotations.
- Try to stick at this stage to rotations of multiples of quarter turns - however, if appropriate you can explore the ideas of rotations of less than or more than a quarter turn for example.
- Within this position and direction work, there is a general path of
It is common for children to rotate a shape and end up with a reflection because they do not realise that the shape should look the same when viewed 'the right way up'.

Children may not always appreciate that there are a range of ways to describe a possible path between two objects.

When turning, some children may lose count of where they started and/or how much (how many quarters) they have turned. They may need to mark the starting point and count as they turn. This can also lead to children believing that right angles must be made from horizontal and vertical lines only.

Key Assessment Checklist

1. I can describe the position of an object using mathematical language.
2. I can describe movement in a straight line using mathematical language.
3. I can say if an object has been rotated a quarter turn, a half turn or a whole turn and identify the direction (clockwise and anticlockwise).
4. I can rotate an object, clockwise or anti-clockwise, using quarter, three-quarters and half turns.
5. I can follow instructions for a particular path using correct mathematical language.
6. I can give instructions for a particular path using correct mathematical language.

Abstraction moving from gross physical movement of the children themselves, to movement of objects, to movement of virtual objects (using ICT) to movement of abstract objects (on paper). Make sure there are opportunities to follow this progression if needed to secure children's skills in both carrying out and describing these movements.

- There is an opportunity to introduce the points of the compass in this unit too.
**Stage 2**

**Unit 15: Measuring and Estimating**

8 learning hours

**Prior Learning**
- compare, describe and solve practical problems for:
  - lengths and heights (e.g., long/short, longer/shorter, tall/short, double/half)
  - mass/weight (e.g., heavy/light, heavier than, lighter than)
  - capacity and volume (e.g., full/empty, more than, less than, half, half full, quarter)
  - time (e.g., quicker, slower, earlier, later)

**Core Learning**
- choose and use appropriate standard units to estimate and measure length/height in any direction (m/cm); mass (kg/g); temperature (°C); capacity (litres/ml) to the nearest appropriate unit, using rulers, scales, thermometers and measuring vessels
- compare and order lengths, mass, volume/capacity and record the results using >, < and =

**Learning Leads to...**
- measure, compare, add and subtract: lengths (m/cm/mm); mass (kg/g); volume/capacity (l/ml)

### Prior Learning

- **Core Learning**
  - measure and begin to record the following:
    - lengths and heights
    - mass/weight
    - capacity and volume
    - time (hours, minutes, seconds)

### Core Learning

- **Learning Leads to...**

#### Working at expected standard
- read scales in divisions of ones, twos, fives and tens in a practical situation where all numbers on the scale are given (e.g. pupil reads a temperature from a thermometer or measures capacities using a measuring jug)

#### Working at greater depth:
- read scales in divisions of ones, twos, fives and tens in a practical situation where not all numbers on the scale are given

### Exemplification

1. (i) Say which unit would be best for measuring:
   - (a) the length of a pencil
   - (b) the mass of a table
   - (c) the capacity of a bucket
   - (d) the length of the playground
   - (e) the temperature of the moon

   (ii) Joe is measuring the mass of a box using some scales

### Vocabulary

<table>
<thead>
<tr>
<th>Exemplification</th>
<th>Vocabulary</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a) the length of a pencil</td>
<td>unit</td>
</tr>
<tr>
<td>(b) the mass of a table</td>
<td>sensible unit</td>
</tr>
<tr>
<td>(c) the capacity of a bucket</td>
<td>measuring equipment</td>
</tr>
<tr>
<td>(d) the length of the playground</td>
<td>measure</td>
</tr>
<tr>
<td>(e) the temperature of the moon</td>
<td>estimate</td>
</tr>
<tr>
<td>(ii) Joe is measuring the mass of a box using some scales</td>
<td>length</td>
</tr>
<tr>
<td></td>
<td>height</td>
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<tr>
<td></td>
<td>width</td>
</tr>
</tbody>
</table>

| (i) Joe is measuring the mass of a box using some scales | gram(s) |
| | litre(s) |
| | centimetre(s) |
| | kilogram(s) |
| | millilitre(s) |
| | prefixes: milli, centi, kilo |
| | together with metres, grams, |
What is the mass of the box?

2. a) Put these lengths in order from smallest to largest
12 cm 12 m 2 cm 2m 212 m

b) Put the signs < or > in to these gaps to make the sentences true
The capacity of a bath …… the capacity of a bucket
The mass of a mouse …… the mass of a human

<table>
<thead>
<tr>
<th>Representation</th>
<th>Fluency</th>
<th>Probing Questions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Measuring Length</td>
<td>1. Measure lengths</td>
<td>Show me a piece of equipment you could use to measure: the width of this classroom, the height of a pencil; the length of the field.</td>
</tr>
<tr>
<td></td>
<td>• measure short length in centimetres using a ruler</td>
<td>Show me a line that is 2cm longer than this one</td>
</tr>
<tr>
<td></td>
<td>• measure a longer length in centimetres using a tape measure, measuring tape or metre rule</td>
<td>Convince me that there will always be more cm in a measurement than m</td>
</tr>
<tr>
<td></td>
<td>• measure a longer length in metres using a metre rule, tape measure or trundle wheel</td>
<td>Convince me that 130cm &gt; 1m</td>
</tr>
<tr>
<td></td>
<td>• record the measurement with the units</td>
<td>What’s the same and what’s different? ruler; tape measure; metre rule; trundle wheel</td>
</tr>
<tr>
<td></td>
<td>• know that there are 100cm in a 1 metre</td>
<td></td>
</tr>
<tr>
<td></td>
<td>• produce a line or other object of a given length using a ruler or tape measure/metre rule.</td>
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<tr>
<td>• Exploring the use of balance scales, electronic scales, scales with a dial scale, hanging balances for measuring masses&lt;br&gt;• Estimating a mass and then checking by measuring&lt;br&gt;• Deciding which is the best piece of equipment for measuring different masses</td>
<td>• measure small masses in grams using a set of scales (electronic and with scale)&lt;br&gt;• measure larger masses in kilograms using a set of scales (electronic and with scale)&lt;br&gt;• record the measurement with the units&lt;br&gt;• know that there are 1000g in a 1 kilogram&lt;br&gt;• gather objects together to produce a given mass using a set of scales</td>
<td>• measure small capacities in millilitres using a set of a measuring jug or measuring cylinder&lt;br&gt;• measure larger capacities in litres using measuring jugs or other 1-litre containers&lt;br&gt;• record the measurement with the units&lt;br&gt;• know that there are 1000ml in a 1 litre&lt;br&gt;• produce an amount of water with a given capacity e.g. 200ml</td>
</tr>
<tr>
<td></td>
<td>Show me an estimate for the weight of this pencil case&lt;br&gt;Show me something that has a mass of approximately a kilogram&lt;br&gt;Show me a unit you could use to measure the mass of this chair&lt;br&gt;Convince me that mass cannot be measured in cm</td>
<td>Show me how you could use this measuring jug to measure the capacity of this cup.&lt;br&gt;Always, Sometimes, Never?&lt;br&gt;You can measure the capacity of a bucket in cups.</td>
</tr>
<tr>
<td>Measuring Capacity</td>
<td>4. Measure other quantities</td>
<td>5. Read and mark scales on equipment correctly</td>
</tr>
<tr>
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</tr>
<tr>
<td>• Exploring the use of measuring cylinders, measuring jugs, litre containers, larger containers and informal measures e.g. cups for measuring capacities&lt;br&gt;• Estimating a capacity and then checking by measuring&lt;br&gt;• Deciding which is the best piece of equipment for measuring different capacities</td>
<td>• temperatures (in °C)&lt;br&gt;• time (in seconds and minutes)</td>
<td>• read a measurement from a scale marked in 1s with all numbers shown&lt;br&gt;• mark a measurement on a scale marked in 1s with all numbers shown&lt;br&gt;• read a measurement from a scale marked in 1s but with only periodic numbers shown e.g. every 5&lt;br&gt;• mark a measurement on a scale marked in 1s but with only periodic numbers shown e.g. every 5&lt;br&gt;• ext: read or mark on a measurement on a scale marked in 2s or 10s or 5s</td>
</tr>
<tr>
<td>Measuring Other Quantities</td>
<td></td>
<td></td>
</tr>
<tr>
<td>• Exploring the use of thermometers for measuring temperature&lt;br&gt;• Exploring the use of stopwatches for measuring time&lt;br&gt;• Estimating a time and then checking by measuring</td>
<td>Convince me that you cannot use a thermometer to measure the height of a cup&lt;br&gt;Always, Sometimes, Never?&lt;br&gt;You can measure any time in seconds</td>
<td></td>
</tr>
<tr>
<td>Scales</td>
<td></td>
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<tr>
<td>• Exploring the scales on apparatus already encountered e.g. kitchen scales, thermometers, rulers to see where different numbers sit&lt;br&gt;• Marking a counting stick in 1s, 2s, 5s or 10s and finding where different measurements would go</td>
<td></td>
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</tr>
</tbody>
</table>
### Units
- Selecting equipment from a table to measure a given measurement and reporting back to the group as to whether the equipment and unit was suitable.

### 6. Suggest a unit for a given measurement
- lengths (widths, heights, lengths and distances)
- masses
- capacities
- temperatures
- times/durations
- say why a unit of measurement may not be suitable (e.g. too small or too larger)

### Comparing and Ordering
- Representing a measurement practically to help compare it e.g. 12cm or 15cm? 2 litres or 200 ml?
- Positioning measurements on a scale to show which is greater and which is less
- Arranging measures on cards or shown practically into order

### 7. Compare measurements using the signs =, < and >
- given two measurements in the same units, say which is greater and show this using < or >
- given two measurements in different units that do not overlap e.g. 12cm and 3m, say which is greater and show this using < or >
- given a measurement made from a combination of smaller measures (e.g. a set of scales with several weights on it), say which total is greater and show this using < or > (or =)
- read two measurements from scales and then compare them
- give a measurement that is greater or less than a stated measurement
- ext: given two measurements in different units that overlap e.g. 120cm and 1m, say which is greater and show this using < or >

### 8. Order a set of measurements
- order three measurements in the same units
- order more than three measurements in the same units
- order measurements in mixed units (no overlaps)
- ext: order measurements in mixed units (with overlaps)

### Questions
- Show me a piece of equipment you could use to measure:
  - the width of this classroom;
  - the weight of your shoe;
  - how long this lesson lasts;
  - how much water a cup holds.
- Convince me that estimates can be good even if they aren't the same as the actual measurement

### What’s the same and what’s different?
- cm, m, g, kg, ml, l, C

### Always, Sometimes, Never?
- Measurements in metres are longer than measurements in centimetres.
- A measuring tool can measure more than one of the following: length, weight, temperature, time.
<table>
<thead>
<tr>
<th>Further Extension</th>
<th>Rich and Sophisticated Tasks</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. What is the mass of two red bags? Which is heavier, the red bag or the green bag? Explain your reasoning.</td>
<td>Choose and use appropriate standard units to estimate and measure length/height in any direction (m/cm); mass (kg/g); temperature (°C); capacity (litres/ml) to the nearest appropriate unit, using rulers, scales, thermometers and measuring vessels</td>
</tr>
<tr>
<td></td>
<td>NRICH: <a href="https://nrich.maths.org/article/1583">Discuss and Choose</a> * G</td>
</tr>
<tr>
<td></td>
<td>NRICH: <a href="https://nrich.maths.org/article/1628">Little Man</a> * P</td>
</tr>
<tr>
<td>2. How long is the crayon?</td>
<td>Compare and order lengths, mass, volume/capacity and record the results using &gt;, &lt; and =</td>
</tr>
<tr>
<td></td>
<td>NRICH: <a href="https://nrich.maths.org/article/1702">Compare the Cups</a> * P</td>
</tr>
</tbody>
</table>
## Misconceptions

Children may muddle up what they are measuring and, for example, try to use a ruler to measure a capacity. They may also fail to see the true difference between measuring and estimating; you may also have a child who believes that their estimate was wrong because it was not exactly the same as the final measurement.

Children may not always measure from zero e.g. they may align an object to the end of a ruler rather than the 0 mark when measuring length or fail to reset the scales when measuring mass.

Some children believe measures using smaller units must be smaller in amount than any using a larger unit e.g. they do not believe that 120cm can be longer than 1m because centimetres are smaller than metres.

Many children find reading scales (especially non-linear ones e.g. dials) very difficult. This is particularly exacerbated when the scale does not go up in single units.

Children do not always read < and > as less than and greater than, instead they may see it as an arrow, for example.

## Teacher Guidance and Notes

- In Stage 2, the focus is on developing fluency and confidence with standard units for length, mass, capacity and temperature.
- The expectations of the specific units of measurements are shown in the National Curriculum statements above - however, since there is a systematic approach to metric units using the prefixes system, you should be prepared for children to want to move outside of this at times. This is actually beneficial as it helps them to understand that there are a wide range of units to measure mass, say, but that only some of them are really suitable for finding the mass of a car, for example.
- Try also to develop a sense of what the most common units look/feel like e.g. how long is a metre roughly? How does 1 kilogram feel?
- Be mindful of spending sufficient time on capacity (and to some extent on mass) as this area is often overlooked due to the more demanding nature of the practical work.
- Part of this unit involves children reading scales from different equipment – at this level this should be with scales that go up in single units but be prepared to extend to scales going up in 2s, 10s and 5s (or even 100s etc) for those working at greater depth
- When making comparisons between amounts, you should use the inequality signs < and >. Encourage children to say less than and greater than as the mathematical translations and encourage them to use this language when they are discussing comparisons. Beware children making comparisons using different units and looking only at the numbers, rather than the units, e.g. saying 2m < 20cm because 2 < 20.

## Key Assessment Checklist

1. I can choose and use appropriate standard units to measure length to the nearest unit (using rulers)
2. I can choose and use appropriate standard units to measure mass to the nearest unit (using scales)
3. I can choose, make estimates and use appropriate standard units to measure capacity to the nearest unit using measuring vessels
4. I can choose, make estimates and use appropriate standard units to measure temperature to the nearest unit (using thermometers)
5. I can read scales on measuring equipment correctly.
6. I make estimates using appropriate standard units to measure length, mass, capacity and temperature.
7. I can compare units and record comparisons using inequality symbols.
8. I can order units and record results